MATHEMATICAL TABLES

J. M. PEIRCE

MATHEMATICAL TABLES

CHIEFLY TO FOUR FIGURES

FIRST SERIES

BY

JAMES MILLS PEIRCE

UNIVERSITY PROFESSOR OF MATHEMATICS IN HARVARD UNIVERSITY

BOSTON, U.S.A.:

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11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14
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17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	5	7	9
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6
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42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4

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55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1. 2. 2. 3. 4
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1. 2. 2 3. 4
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1. 2. 2. 3. 4
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1. 1. 2. 3. 4
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1. 1. 2. 3 4
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1. 1. 2. 3. 4
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1. 1. 2. 3. 4
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1. 2. 3. 3
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1. 1. 2. 3. 3
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1. 1. 2. 3. 3
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1. 1 2. 3. 3
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1. 1. 2. 3. 3
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1. 1. 2. 3. 3
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1. 1. 2. 3. 3
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1. 1. 2. 3. 3
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1. 1. 2. 2. 3
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1. 1. 2. 2. 3
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1. 1. 2. 2. 3
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1. 1. 2. 2. 3
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1. 1. 2. 2. 3
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1. 1. 2. 2. 3
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1. 1. 2. 2. 3
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1. 1. 2. 2. 3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1. 1. 2. 2. 3
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1. 1. 2. 2. 3
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1. 1. 2. 2. 3
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1. 1. 2. 2. 3
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1. 1. 2. 2. 3
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1. 1. 2. 2. 3
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1. 1. 2. 2. 3
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1. 1. 2. 2. 3
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1. 1. 2. 2. 3
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0. 1. 1. 2. 2
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0. 1. 1. 2. 2
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0. 1. 1. 2. 2
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0. 1. 1. 2. 2
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0. 1. 1. 2. 2
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0. 1. 1. 2. 2
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0. 1. 1. 2. 2
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1. 1. 2. 2
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0. 1. 1. 2. 2
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0. 1. 1. 2. 2
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0. 1. 1. 2. 2
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1. 1. 2. 2

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101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	0086
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	0212
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	0253
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	0569
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	0607
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	0645
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	0719
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	0755
119	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	0792
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	0828
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	0899
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	1038
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	1072
128	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	1106
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	1139
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	1173
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	1206
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	1239
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	1271
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	1303
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	1335
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	1367
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	1399
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	1430
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	1461
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	1492
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	1523
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	1553
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	1584
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	1614
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	1644
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	1673
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	1703
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	1732
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	1761

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150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	1790
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	1818
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	2014
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	2068
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	2095
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	2253
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	2279
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	2330
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	2355
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	2380
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	2405
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2430
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	2455
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	2504
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	2577
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	2601
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	2625
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	2648
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
194	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

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00	0.0000	0.0004	0.0043	0.0414 9	0.3010 50	1.0414 91	2.0043	3.0004
01	0.0000	0.0004	0.0044	0.0423 9	0.3061 51	1.0505 91	2.0142	3.0104
02	0.0000	0.0005	0.0045	0.0432 9	0.3111 51	1.0596 91	2.0241	3.0204
03	0.0000	0.0005	0.0046	0.0442 10	0.3163 52	1.0687 91	2.0340	3.0304
04	0.0000	0.0005	0.0047	0.0452 10	0.3215 52	1.0779 92	2.0439	3.0404
05	0.0000	0.0005	0.0048	0.0462 10	0.3267 53	1.0871 92	2.0539	3.0504
06	0.0000	0.0005	0.0050	0.0472 10	0.3321 53	1.0963 92	2.0638	3.0604
07	0.0001	0.0005	0.0051	0.0482 11	0.3374 54	1.1055 92	2.0737	3.0704
08	0.0001	0.0005	0.0052	0.0493 11	0.3429 55	1.1147 92	2.0836	3.0804
09	0.0001	0.0005	0.0053	0.0504 11	0.3484 55	1.1239 92	2.0935	3.0904
10	0.0001	0.0005	0.0054	0.0515 11	0.3539 56	1.1332 93	2.1034	3.1003
11	0.0001	0.0006	0.0056	0.0526 11	0.3595 56	1.1425 93	2.1134	3.1103
12	0.0001	0.0006	0.0057	0.0538 12	0.3652 57	1.1518 93	2.1233	3.1203
13	0.0001	0.0006	0.0058	0.0550 12	0.3709 57	1.1611 93	2.1332	3.1303
14	0.0001	0.0006	0.0060	0.0562 12	0.3766 58	1.1704 93	2.1431	3.1403
15	0.0001	0.0006	0.0061	0.0574 12	0.3825 59	1.1797 93	2.1531	3.1503
16	0.0001	0.0006	0.0062	0.0586 13	0.3884 59	1.1891 94	2.1630	3.1603
17	0.0001	0.0006	0.0064	0.0599 13	0.3943 60	1.1984 94	2.1729	3.1703
18	0.0001	0.0007	0.0065	0.0612 13	0.4003 60	1.2078 94	2.1829	3.1803
19	0.0001	0.0007	0.0067	0.0625 13	0.4063 61	1.2172 94	2.1928	3.1903
20	0.0001	0.0007	0.0068	0.0639 14	0.4124 61	1.2266 94	2.2027	3.2003
21	0.0001	0.0007	0.0070	0.0653 14	0.4186 62	1.2360 94	2.2127	3.2103
22	0.0001	0.0007	0.0071	0.0667 14	0.4248 62	1.2454 94	2.2226	3.2203
23	0.0001	0.0007	0.0073	0.0681 15	0.4311 63	1.2548 94	2.2325	3.2303
24	0.0001	0.0008	0.0075	0.0696 15	0.4374 63	1.2643 95	2.2425	3.2402
25	0.0001	0.0008	0.0077	0.0711 15	0.4438 64	1.2738 95	2.2524	3.2502
26	0.0001	0.0008	0.0078	0.0726 15	0.4502 65	1.2832 95	2.2624	3.2602
27	0.0001	0.0008	0.0080	0.0742 16	0.4567 65	1.2927 95	2.2723	3.2702
28	0.0001	0.0008	0.0082	0.0757 16	0.4632 66	1.3022 95	2.2823	3.2802
29	0.0001	0.0008	0.0084	0.0774 16	0.4698 66	1.3117 95	2.2922	3.2902
30	0.0001	0.0009	0.0086	0.0790 17	0.4764 67	1.3212 95	2.3022	3.3002
31	0.0001	0.0009	0.0088	0.0807 17	0.4831 67	1.3308 95	2.3121	3.3102
32	0.0001	0.0009	0.0090	0.0824 17	0.4899 68	1.3403 95	2.3221	3.3202
33	0.0001	0.0009	0.0092	0.0841 18	0.4966 68	1.3499 96	2.3320	3.3302
34	0.0001	0.0009	0.0094	0.0859 18	0.5035 69	1.3594 96	2.3420	3.3402
35	0.0001	0.0010	0.0096	0.0877 18	0.5104 69	1.3690 96	2.3519	3.3502
36	0.0001	0.0010	0.0098	0.0896 19	0.5173 70	1.3786 96	2.3619	3.3602
37	0.0001	0.0010	0.0101	0.0915 19	0.5243 70	1.3881 96	2.3718	3.3702
38	0.0001	0.0010	0.0103	0.0934 19	0.5313 71	1.3977 96	2.3818	3.3802
39	0.0001	0.0011	0.0105	0.0953 20	0.5384 71	1.4073 96	2.3918	3.3902
40	0.0001	0.0011	0.0108	0.0973 20	0.5455 72	1.4170 96	2.4017	3.4002
41	0.0001	0.0011	0.0110	0.0993 20	0.5527 72	1.4266 96	2.4117	3.4102
42	0.0001	0.0011	0.0113	0.1014 21	0.5599 72	1.4362 96	2.4216	3.4202
43	0.0001	0.0012	0.0115	0.1035 21	0.5672 73	1.4458 96	2.4316	3.4302
44	0.0001	0.0012	0.0118	0.1057 22	0.5745 73	1.4555 96	2.4416	3.4402
45	0.0001	0.0012	0.0121	0.1078 22	0.5819 74	1.4651 97	2.4515	3.4502
46	0.0001	0.0013	0.0123	0.1101 22	0.5893 74	1.4748 97	2.4615	3.4602
47	0.0001	0.0013	0.0126	0.1123 23	0.5967 75	1.4845 97	2.4715	3.4701
48	0.0001	0.0013	0.0129	0.1146 23	0.6042 75	1.4941 97	2.4814	3.4801
49	0.0001	0.0013	0.0132	0.1169 24	0.6118 76	1.5038 97	2.4914	3.4901
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001

A	6.	7.	8.	9.	0.	1.	2.	3.
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001
51	0.0001	0.0014	0.0138	0.1218 24	0.6269 76	1.5232 97	2.5113	3.5101
52	0.0001	0.0014	0.0141	0.1242 25	0.6346 77	1.5329 97	2.5213	3.5201
53	0.0001	0.0015	0.0145	0.1267 25	0.6423 77	1.5426 97	2.5313	3.5301
54	0.0002	0.0015	0.0148	0.1293 26	0.6501 78	1.5523 97	2.5413	3.5401
55	0.0002	0.0015	0.0151	0.1319 26	0.6578 78	1.5621 97	2.5512	3.5501
56	0.0002	0.0016	0.0155	0.1345 27	0.6657 78	1.5718 97	2.5612	3.5601
57	0.0002	0.0016	0.0158	0.1372 27	0.6735 79	1.5815 97	2.5712	3.5701
58	0.0002	0.0016	0.0162	0.1399 28	0.6814 79	1.5913 97	2.5811	3.5801
59	0.0002	0.0017	0.0166	0.1427 28	0.6893 80	1.6010 97	2.5911	3.5901
60	0.0002	0.0017	0.0170	0.1455 28	0.6973 80	1.6108 98	2.6011	3.6001
61	0.0002	0.0018	0.0173	0.1484 29	0.7053 80	1.6205 98	2.6111	3.6101
62	0.0002	0.0018	0.0177	0.1513 29	0.7134 81	1.6303 98	2.6210	3.6201
63	0.0002	0.0018	0.0181	0.1543 30	0.7215 81	1.6401 98	2.6310	3.6301
64	0.0002	0.0019	0.0186	0.1573 30	0.7296 81	1.6498 98	2.6410	3.6401
65	0.0002	0.0019	0.0190	0.1604 31	0.7377 82	1.6596 98	2.6510	3.6501
66	0.0002	0.0020	0.0194	0.1635 31	0.7459 82	1.6694 98	2.6609	3.6601
67	0.0002	0.0020	0.0199	0.1666 32	0.7541 82	1.6792 98	2.6709	3.6701
68	0.0002	0.0021	0.0203	0.1699 32	0.7624 83	1.6890 98	2.6809	3.6801
69	0.0002	0.0021	0.0208	0.1731 33	0.7707 83	1.6988 98	2.6909	3.6901
70	0.0002	0.0022	0.0212	0.1764 33	0.7790 83	1.7086 98	2.7009	3.7001
71	0.0002	0.0022	0.0217	0.1798 34	0.7874 84	1.7184 98	2.7108	3.7101
72	0.0002	0.0023	0.0222	0.1832 34	0.7957 84	1.7282 98	2.7208	3.7201
73	0.0002	0.0023	0.0227	0.1867 35	0.8042 84	1.7380 98	2.7308	3.7301
74	0.0002	0.0024	0.0232	0.1902 35	0.8126 85	1.7478 98	2.7408	3.7401
75	0.0002	0.0024	0.0238	0.1938 36	0.8211 85	1.7577 98	2.7508	3.7501
76	0.0002	0.0025	0.0243	0.1974 37	0.8296 85	1.7675 98	2.7608	3.7601
77	0.0003	0.0025	0.0248	0.2011 37	0.8381 85	1.7773 98	2.7707	3.7701
78	0.0003	0.0026	0.0254	0.2048 38	0.8467 86	1.7871 98	2.7807	3.7801
79	0.0003	0.0027	0.0260	0.2086 38	0.8553 86	1.7970 98	2.7907	3.7901
80	0.0003	0.0027	0.0266	0.2124 39	0.8639 86	1.8068 98	2.8007	3.8001
81	0.0003	0.0028	0.0272	0.2163 39	0.8725 87	1.8167 98	2.8107	3.8101
82	0.0003	0.0029	0.0278	0.2203 40	0.8812 87	1.8265 99	2.8207	3.8201
83	0.0003	0.0029	0.0284	0.2243 40	0.8899 87	1.8364 99	2.8306	3.8301
84	0.0003	0.0030	0.0291	0.2284 41	0.8986 87	1.8462 99	2.8406	3.8401
85	0.0003	0.0031	0.0297	0.2325 41	0.9074 88	1.8561 99	2.8506	3.8501
86	0.0003	0.0031	0.0304	0.2366 42	0.9162 88	1.8660 99	2.8606	3.8601
87	0.0003	0.0032	0.0311	0.2409 43	0.9250 88	1.8758 99	2.8706	3.8701
88	0.0003	0.0033	0.0318	0.2452 43	0.9338 88	1.8857 99	2.8806	3.8801
89	0.0003	0.0034	0.0325	0.2495 44	0.9426 89	1.8956 99	2.8906	3.8901
90	0.0003	0.0034	0.0332	0.2539 44	0.9515 89	1.9054 99	2.9005	3.9001
91	0.0004	0.0035	0.0339	0.2584 45	0.9604 89	1.9153 99	2.9105	3.9101
92	0.0004	0.0036	0.0347	0.2629 45	0.9693 89	1.9252 99	2.9205	3.9201
93	0.0004	0.0037	0.0355	0.2674 46	0.9782 89	1.9351 99	2.9305	3.9301
94	0.0004	0.0038	0.0363	0.2721 47	0.9872 90	1.9450 99	2.9405	3.9400
95	0.0004	0.0039	0.0371	0.2767 47	0.9962 90	1.9548 99	2.9505	3.9500
96	0.0004	0.0039	0.0379	0.2815 48	1.0052 90	1.9647 99	2.9605	3.9600
97	0.0004	0.0040	0.0387	0.2863 48	1.0142 90	1.9746 99	2.9705	3.9700
98	0.0004	0.0041	0.0396	0.2911 49	1.0232 91	1.9845 99	2.9805	3.9800
99	0.0004	0.0042	0.0405	0.2961 49	1.0323 91	1.9944 99	2.9904	3.9900
00	0.0004	0.0043	0.0414	0.3010 50	1.0414 91	2.0043 99	3.0004	4.0000

φ	0° lsin ltn	lsc	1° lsin ltn	lsc	2° lsin ltn	lsc	
00′	— ∞	00	8.2419 19 72	01	8.5428 31 36	03	60′
01′	6.4637 37	00	8.2490 91 71	01	8.5464 67 36	03	59′
02′	6.7648 48	00	8.2561 62 70	01	8.5500 03 36	03	58′
03′	6.9408 08	00	8.2630 31 69	01	8.5535 38 35	03	57′
04′	7.0658 58	00	8.2699*00 68	01	8.5571 73 35	03	56′
05′	7.1627 27	00	8.2766 67 67	01	8.5605 08 35	03	55′
06′	7.2419 19	00	8.2832 33 66	01	8.5640 43 34	03	54′
07′	7.3088 88	00	8.2898 99 65	01	8.5674 77 34	03	53′
08′	7.3668 68	00	8.2962 63 64	01	8.5708 11 34	03	52′
09′	7.4180 80	00	8.3025 26 63	01	8.5742 45 34	03	51′
10′	7.4637 37	00	8.3088 89 62	01	8.5776 79 33	03	50′
11′	7.5051 51	00	8.3150 50 61	01	8.5809 12 33	03	49′
12′	7.5429 29	00	8.3210 11 60	01	8.5842 45 33	03	48′
13′	7.5777 77	00	8.3270 71 59	01	8.5875 78 33	03	47′
14′	7.6099 99	00	8.3329 30 59	01	8.5907 11 32	03	46′
15′	7.6398 98	00	8.3388 89 58	01	8.5939 43 32	03	45′
16′	7.6678 78	00	8.3445 46 57	01	8.5972 75 32	03	44′
17′	7.6942 42	00	8.3502 03 56	01	8.6003 07 32	03	43′
18′	7.7190 90	00	8.3558 59 56	01	8.6035 38 31	04	42′
19′	7.7425 25	00	8.3613 14 55	01	8.6066 70 31	04	41′
20′	7.7648 48	00	8.3668 69 54	01	8.6097*01 31	04	40′
21′	7.7859 60	00	8.3722 23 54	01	8.6128 32 31	04	39′
22′	7.8061 62	00	8.3775 76 53	01	8.6159 63 31	04	38′
23′	7.8255 55	00	8.3828 29 52	01	8.6189 93 30	04	37′
24′	7.8439 39	00	8.3880 81 52	01	8.6220 23 30	04	36′
25′	7.8617 17	00	8.3931 32 51	01	8.6250 54 30	04	35′
26′	7.8787 87	00	8.3982 83 51	01	8.6279 83 30	04	34′
27′	7.8951 51	00	8.4032 33 50	01	8.6309 13 30	04	33′
28′	7.9109 09	00	8.4082 83 49	01	8.6339 43 29	04	32′
29′	7.9261 61	00	8.4131 32 49	01	8.6368 72 29	04	31′
30′	7.9408 09	00	8.4179 81 48	01	8.6397*01 29	04	30′
31′	7.9551 51	00	8.4227 29 48	02	8.6426 30 29	04	29′
32′	7.9689 89	00	8.4275 76 47	02	8.6454 59 29	04	28′
33′	7.9822 23 132	00	8.4322 23 47	02	8.6483 87 28	04	27′
34′	7.9952 52 128	00	8.4368 70 46	02	8.6511 15 28	04	26′
35′	8.0078 78 124	00	8.4414 16 46	02	8.6539 44 28	04	25′
36′	8.0200 00 121	00	8.4459 61 45	02	8.6567 71 28	04	24′
37′	8.0319 19 117	00	8.4504 06 45	02	8.6595 99 28	05	23′
38′	8.0435 35 114	00	8.4549 51 44	02	8.6622 27 27	05	22′
39′	8.0548 48 111	00	8.4593 95 44	02	8.6650 54 27	05	21′
40′	8.0658 58 109	00	8.4637 38 43	02	8.6677 82 27	05	20′
41′	8.0765 65 106	00	8.4680 82 43	02	8.6704 09 27	05	19′
42′	8.0870 70 103	00	8.4723 25 43	02	8.6731 36 27	05	18′
43′	8.0972 72 101	00	8.4765 67 42	02	8.6758 62 27	05	17′
44′	8.1072 72 99	00	8.4807 09 42	02	8.6784 89 26	05	16′
45′	8.1169 70 97	00	8.4848 51 41	02	8.6810 15 26	05	15′
46′	8.1265 65 94	00	8.4890 92 41	02	8.6837 42 26	05	14′
47′	8.1358 59 92	00	8.4930 33 41	02	8.6863 68 26	05	13′
48′	8.1450 50 90	00	8.4971 73 40	02	8.6889 94 26	05	12′
49′	8.1539 40 89	00	8.5011 13 40	02	8.6914 20 26	05	11′
50′	8.1627 27 87	00	8.5050 53 39	02	8.6940 45 26	05	10′
51′	8.1713 13 85	00	8.5090 92 39	02	8.6965 71 25	05	09′
52′	8.1797 98 84	00	8.5129 31 39	02	8.6991 96 25	05	08′
53′	8.1880 80 82	01	8.5167 70 38	02	8.7016 21 25	06	07′
54′	8.1961 62 80	01	8.5206 08 38	02	8.7041 46 25	06	06′
55′	8.2041 41 79	01	8.5243 46 38	02	8.7066 71 25	06	05′
56′	8.2119 20 78	01	8.5281 83 37	02	8.7090 96 25	06	04′
57′	8.2196 96 76	01	8.5318 21 37	03	8.7115 21 25	06	03′
58′	8.2271 72 75	01	8.5355 58 37	03	8.7140 45 24	06	02′
59′	8.2346 46 74	01	8.5392 94 37	03	8.7164 70 24	06	01′
60′	8.2419 19 72	01	8.5428 31 36	03	8.7188 94 24	06	00′
	89° lcos lctn	lcsc	88° lcos lctn	lcsc	87° lcos lctn	lcsc	θ

φ	3° lsin ltn	lsc	4° lsin ltn	lsc	5° lsin ltn	lsc	
00′	8.7188 94 24	06	8.8436 46 18	11	8.9403 20 14	17	60′
01′	8.7212 18 24	06	8.8454 65 18	11	8.9417 34 14	17	59′
02′	8.7236 42 24	06	8.8472 83 18	11	8.9432 49 14	17	58′
03′	8.7260 66 24	06	8.8490*01 18	11	8.9446 63 14	17	57′
04′	8.7283 90 24	06	8.8508 18 18	11	8.9460 77 14	17	56′
05′	8.7307 13 23	06	8.8525 36 18	11	8.9475 92 14	17	55′
06′	8.7330 37 23	06	8.8543 54 18	11	8.9489*06 14	17	54′
07′	8.7354 60 23	06	8.8560 72 18	11	8.9503 20 14	17	53′
08′	8.7377 83 23	06	8.8578 89 18	11	8.9517 34 14	17	52′
09′	8.7400 06 23	07	8.8595*07 17	11	8.9531 49 14	18	51′
10′	8.7423 29 23	07	8.8613 24 17	11	8.9545 63 14	18	50′
11′	8.7445 52 23	07	8.8630 42 17	12	8.9559 77 14	18	49′
12′	8.7468 75 23	07	8.8647 59 17	12	8.9573 91 14	18	48′
13′	8.7491 97 23	07	8.8665 76 17	12	8.9587*05 14	18	47′
14′	8.7513 20 22	07	8.8682 94 17	12	8.9601 19 14	18	46′
15′	8.7535 42 22	07	8.8699*11 17	12	8.9614 33 14	18	45′
16′	8.7557 65 22	07	8.8716 28 17	12	8.9628 46 14	18	44′
17′	8.7580 87 22	07	8.8733 45 17	12	8.9642 60 14	18	43′
18′	8.7602 09 22	07	8.8749 62 17	12	8.9655 74 14	19	42′
19′	8.7623 31 22	07	8.8766 78 17	12	8.9669 88 14	19	41′
20′	8.7645 52 22	07	8.8783 95 17	12	8.9682*01 14	19	40′
21′	8.7667 74 22	07	8.8799*12 17	13	8.9696*15 14	19	39′
22′	8.7688 96 22	08	8.8816 29 17	13	8.9709 29 14	19	38′
23′	8.7710 17 21	08	8.8833 45 17	13	8.9723 42 13	19	37′
24′	8.7731 39 21	08	8.8849 62 16	13	8.9736 56 13	19	36′
25′	8.7752 60 21	08	8.8865 78 16	13	8.9750 69 13	19	35′
26′	8.7773 81 21	08	8.8882 95 16	13	8.9763 82 13	20	34′
27′	8.7794*02 21	08	8.8898*11 16	13	8.9776 96 13	20	33′
28′	8.7815 23 21	08	8.8914 27 16	13	8.9789*09 13	20	32′
29′	8.7836 44 21	08	8.8930 44 16	13	8.9803 23 13	20	31′
30′	8.7857 65 21	08	8.8946 60 16	13	8.9816 36 13	20	30′
31′	8.7877 86 21	08	8.8962 76 16	14	8.9829 49 13	20	29′
32′	8.7898*06 20	08	8.8978 92 16	14	8.9842 62 13	20	28′
33′	8.7918 27 20	08	8.8994*08 16	14	8.9855 75 13	20	27′
34′	8.7939 47 20	08	8.9010 24 16	14	8.9868 88 13	21	26′
35′	8.7959 67 20	08	8.9026 40 16	14	8.9881*01 13	21	25′
36′	8.7979 88 20	09	8.9042 56 16	14	8.9894*15 13	21	24′
37′	8.7999*08 20	09	8.9057 71 16	14	8.9907 28 13	21	23′
38′	8.8019 28 20	09	8.9073 87 16	14	8.9919 40 13	21	22′
39′	8.8039 48 20	09	8.9089*03 16	14	8.9932 53 13	21	21′
40′	8.8059 67 20	09	8.9104 18 16	14	8.9945 66 13	21	20′
41′	8.8078 87 20	09	8.9119 34 15	15	8.9958 79 13	21	19′
42′	8.8098*07 20	09	8.9135 50 15	15	8.9970 92 13	22	18′
43′	8.8117 26 19	09	8.9150 65 15	15	8.9983*05 13	22	17′
44′	8.8137 46 19	09	8.9166 80 15	15	8.9996*17 13	22	16′
45′	8.8156 65 19	09	8.9181 96 15	15	9.0008 30 13	22	15′
46′	8.8175 85 19	09	8.9196*11 15	15	9.0021 43 13	22	14′
47′	8.8194*04 19	09	8.9211 26 15	15	9.0033 55 13	22	13′
48′	8.8213 23 19	10	8.9226 41 15	15	9.0046 68 12	22	12′
49′	8.8232 42 19	10	8.9241 56 15	15	9.0058 80 12	22	11′
50′	8.8251 61 19	10	8.9256 72 15	15	9.0070 93 12	23	10′
51′	8.8270 80 19	10	8.9271 87 15	16	9.0083*05 12	23	09′
52′	8.8289 99 19	10	8.9286*02 15	16	9.0095*18 12	23	08′
53′	8.8307 17 19	10	8.9301 16 15	16	9.0107 30 12	23	07′
54′	8.8326 36 19	10	8.9315 31 15	16	9.0120 43 12	23	06′
55′	8.8345 55 19	10	8.9330 46 15	16	9.0132 55 12	23	05′
56′	8.8363 73 18	10	8.9345 61 15	16	9.0144 67 12	23	04′
57′	8.8381 92 18	10	8.9359 76 15	16	9.0156 80 12	23	03′
58′	8.8400 10 18	10	8.9374 90 15	16	9.0168 92 12	24	02′
59′	8.8418 28 18	11	8.9388*05 15	16	9.0180*04 12	24	01′
60′	8.8436 46 18	11	8.9403 20 14	17	9.0192*16 12	24	00′
	86° lcos lctn	lcsc	85° lcos lctn	lcsc	84° lcos lctn	lcsc	θ

φ	S	l sin φ
	6.46	
0°00′.000		−∞
	37	
1°51′.479		8.5108
	36	
2°49′.567		8.6929
	35	
3°32′.313		8.7904
	34	
4°07′.789		8.8574
	33	
4°38′.783		8.9085
	32	
5°06′.659		8.9498
	31	
5°32′.201		8.9845
	30	
5°55′.913		9.0143
	29	
6°18′.138		9.0405

φ	T	l tan φ
	6.46	
0°00′.000		−∞
	37	
0°44′.155		8.1087
	38	
1°40′.555		8.4663
	39	
2°15′.168		8.5948
	40	
2°42′.563		8.6751
	41	
3°05′.959		8.7336
	42	
3°26′.717		8.7796
	43	
3°45′.567		8.8176
	44	
4°02′.954		8.8500
	45	
4°19′.171		8.8781
	46	
4°34′.427		8.9031
	47	
4°48′.875		8.9255
	48	
5°02′.628		8.9458
	49	
5°15′.780		8.9643
	50	
5°28′.401		8.9815
	51	
5°40′.550		8.9973
	52	
5°52′.278		9.0121
	53	
6°03′.623		9.0260

φ	l sec φ
	0.00
0°00′.000	
	00
0°52′.164	
	01
1°30′.348	
	02
1°56′.634	
	03
2°17′.998	
	04
2°36′.469	
	05
2°52′.976	
	06
3°08′.038	
	07
3°21′.977	
	08
3°35′.016	
	09
3°47′.300	
	10
3°58′.955	
	11
4°10′.064	
	12
4°20′.701	

φ	l sec φ
	0.00
4°20′.701	
	13
4°30′.918	
	14
4°40′.762	
	15
4°50′.271	
	16
4°59′.477	
	17
5°08′.407	
	18
5°17′.084	
	19
5°25′.528	
	20
5°33′.758	
	21
5°41′.789	
	22
5°49′.633	
	23
5°57′.305	
	24
6°04′.814	
	25
6°12′.170	

φ	l sin φ		l csc φ	l tan φ		l ctn φ	l sec φ		l cos φ	
6° 00′	9.0192	120	0.9808	9.0216	122	0.9784	0.0024	1	9.9976	**84° 00′**
6° 10′	9.0311	117	0.9689	9.0336	118	0.9664	0.0025	1	9.9975	83° 50′
6° 20′	9.0426	114	0.9574	9.0453	115	0.9547	0.0027	1	9.9973	83° 40′
6° 30′	9.0539	111	0.9461	9.0567	112	0.9433	0.0028	1	9.9972	83° 30′
6° 40′	9.0648	108	0.9352	9.0678	110	0.9322	0.0029	1	9.9971	83° 20′
6° 50′	9.0755	105	0.9245	9.0786	107	0.9214	0.0031	2	9.9969	83° 10′
7° 00′	9.0859	103	0.9141	9.0891	104	0.9109	0.0032	2	9.9968	**83° 00′**
7° 10′	9.0961	100	0.9039	9.0995	102	0.9005	0.0034	2	9.9966	82° 50′
7° 20′	9.1060	98	0.8940	9.1096	100	0.8904	0.0036	2	9.9964	82° 40′
7° 30′	9.1157	96	0.8843	9.1194	98	0.8806	0.0037	2	9.9963	82° 30′
7° 40′	9.1252	94	0.8748	9.1291	96	0.8709	0.0039	2	9.9961	82° 20′
7° 50′	9.1345	92	0.8655	9.1385	94	0.8615	0.0041	2	9.9959	82° 10′
8° 00′	9.1436	90	0.8564	9.1478	92	0.8522	0.0042	2	9.9958	**82° 00′**
8° 10′	9.1525	88	0.8475	9.1569	90	0.8431	0.0044	2	9.9956	81° 50′
8° 20′	9.1612	86	0.8388	9.1658	88	0.8342	0.0046	2	9.9954	81° 40′
8° 30′	9.1697	85	0.8303	9.1745	86	0.8255	0.0048	2	9.9952	81° 30′
8° 40′	9.1781	83	0.8219	9.1831	85	0.8169	0.0050	2	9.9950	81° 20′
8° 50′	9.1863	81	0.8137	9.1915	83	0.8085	0.0052	2	9.9948	81° 10′
9° 00′	9.1943	80	0.8057	9.1997	82	0.8003	0.0054	2	9.9946	**81° 00′**
9° 10′	9.2022	78	0.7978	9.2078	80	0.7922	0.0056	2	9.9944	80° 50′
9° 20′	9.2100	77	0.7900	9.2158	79	0.7842	0.0058	2	9.9942	80° 40′
9° 30′	9.2176	75	0.7824	9.2236	78	0.7764	0.0060	2	9.9940	80° 30′
9° 40′	9.2251	74	0.7749	9.2313	76	0.7687	0.0062	2	9.9938	80° 20′
9° 50′	9.2324	73	0.7676	9.2389	75	0.7611	0.0064	2	9.9936	80° 10′
10° 00′	9.2397	72	0.7603	9.2463	74	0.7537	0.0066	2	9.9934	**80° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

φ	l sin φ		l csc φ	l tan φ		l ctn φ	l sec φ		l cos φ	
10° 00′	9.2397	72	0.7603	9.2463	74	0.7537	0.0066	2	9.9934	**80° 00′**
10° 10′	9.2468	70	0.7532	9.2536	73	0.7464	0.0069	2	9.9931	79° 50′
10° 20′	9.2538	69	0.7462	9.2609	72	0.7391	0.0071	2	9.9929	79° 40′
10° 30′	9.2606	68	0.7394	9.2680	71	0.7320	0.0073	2	9.9927	79° 30′
10° 40′	9.2674	67	0.7326	9.2750	69	0.7250	0.0076	2	9.9924	79° 20′
10° 50′	9.2740	66	0.7260	9.2819	68	0.7181	0.0078	2	9.9922	79° 10′
11° 00′	9.2806	65	0.7194	9.2887	67	0.7113	0.0081	2	9.9919	**79° 00′**
11° 10′	9.2870	64	0.7130	9.2953	66	0.7047	0.0083	2	9.9917	78° 50′
11° 20′	9.2934	63	0.7066	9.3020	66	0.6980	0.0086	3	9.9914	78° 40′
11° 30′	9.2997	62	0.7003	9.3085	65	0.6915	0.0088	3	9.9912	78° 30′
11° 40′	9.3058	61	0.6942	9.3149	64	0.6851	0.0091	3	9.9909	78° 20′
11° 50′	9.3119	60	0.6881	9.3212	63	0.6788	0.0093	3	9.9907	78° 10′
12° 00′	9.3179	59	0.6821	9.3275	62	0.6725	0.0096	3	9.9904	**78° 00′**
12° 10′	9.3238	59	0.6762	9.3336	61	0.6664	0.0099	3	9.9901	77° 50′
12° 20′	9.3296	58	0.6704	9.3397	61	0.6603	0.0101	3	9.9899	77° 40′
12° 30′	9.3353	57	0.6647	9.3458	60	0.6542	0.0104	3	9.9896	77° 30′
12° 40′	9.3410	56	0.6590	9.3517	59	0.6483	0.0107	3	9.9893	77° 20′
12° 50′	9.3466	55	0.6534	9.3576	58	0.6424	0.0110	3	9.9890	77° 10′
13° 00′	9.3521	55	0.6479	9.3634	58	0.6366	0.0113	3	9.9887	**77° 00′**
13° 10′	9.3575	54	0.6425	9.3691	57	0.6309	0.0116	3	9.9884	76° 50′
13° 20′	9.3629	53	0.6371	9.3748	56	0.6252	0.0119	3	9.9881	76° 40′
13° 30′	9.3682	53	0.6318	9.3804	56	0.6196	0.0122	3	9.9878	76° 30′
13° 40′	9.3734	52	0.6266	9.3859	55	0.6141	0.0125	3	9.9875	76° 20′
13° 50′	9.3786	51	0.6214	9.3914	54	0.6086	0.0128	3	9.9872	76° 10′
14° 00′	9.3837	51	0.6163	9.3968	54	0.6032	0.0131	3	9.9869	**76° 00′**
14° 10′	9.3887	50	0.6113	9.4021	53	0.5979	0.0134	3	9.9866	75° 50′
14° 20′	9.3937	49	0.6063	9.4074	53	0.5926	0.0137	3	9.9863	75° 40′
14° 30′	9.3986	49	0.6014	9.4127	52	0.5873	0.0141	3	9.9859	75° 30′
14° 40′	9.4035	48	0.5965	9.4178	52	0.5822	0.0144	3	9.9856	75° 20′
14° 50′	9.4083	48	0.5917	9.4230	51	0.5770	0.0147	3	9.9853	75° 10′
15° 00′	9.4130	47	0.5870	9.4281	51	0.5719	0.0151	3	9.9849	**75° 00′**
15° 10′	9.4177	47	0.5823	9.4331	50	0.5669	0.0154	3	9.9846	74° 50′
15° 20′	9.4223	46	0.5777	9.4381	50	0.5619	0.0157	3	9.9843	74° 40′
15° 30′	9.4269	46	0.5731	9.4430	49	0.5570	0.0161	4	9.9839	74° 30′
15° 40′	9.4314	45	0.5686	9.4479	49	0.5521	0.0164	4	9.9836	74° 20′
15° 50′	9.4359	45	0.5641	9.4527	48	0.5473	0.0168	4	9.9832	74° 10′
16° 00′	9.4403	44	0.5597	9.4575	48	0.5425	0.0172	4	9.9828	**74° 00′**
16° 10′	9.4447	44	0.5553	9.4622	47	0.5378	0.0175	4	9.9825	73° 50′
16° 20′	9.4491	43	0.5509	9.4669	47	0.5331	0.0179	4	9.9821	73° 40′
16° 30′	9.4533	43	0.5467	9.4716	46	0.5284	0.0183	4	9.9817	73° 30′
16° 40′	9.4576	42	0.5424	9.4762	46	0.5238	0.0186	4	9.9814	73° 20′
16° 50′	9.4618	42	0.5382	9.4808	46	0.5192	0.0190	4	9.9810	73° 10′
17° 00′	9.4659	41	0.5341	9.4853	45	0.5147	0.0194	4	9.9806	**73° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

φ	l sin φ		l csc φ	l tan φ		l ctn φ	l sec φ		l cos φ	
17° 00′	9.4659	41	0.5341	9.4853	45	0.5147	0.0194	4	9.9806	**73° 00′**
17° 10′	9.4700	41	0.5300	9.4898	45	0.5102	0.0198	4	9.9802	72° 50′
17° 20′	9.4741	40	0.5259	9.4943	44	0.5057	0.0202	4	9.9798	72° 40′
17° 30′	9.4781	40	0.5219	9.4987	44	0.5013	0.0206	4	9.9794	72° 30′
17° 40′	9.4821	40	0.5179	9.5031	44	0.4969	0.0210	4	9.9790	72° 20′
17° 50′	9.4861	39	0.5139	9.5075	43	0.4925	0.0214	4	9.9786	72° 10′
18° 00′	9.4900	39	0.5100	9.5118	43	0.4882	0.0218	4	9.9782	**72° 00′**
18° 10′	9.4939	38	0.5061	9.5161	43	0.4839	0.0222	4	9.9778	71° 50′
18° 20′	9.4977	38	0.5023	9.5203	42	0.4797	0.0226	4	9.9774	71° 40′
18° 30′	9.5015	38	0.4985	9.5245	42	0.4755	0.0230	4	9.9770	71° 30′
18° 40′	9.5052	37	0.4948	9.5287	42	0.4713	0.0235	4	9.9765	71° 20′
18° 50′	9.5090	37	0 4910	9.5329	41	0.4671	0.0239	4	9.9761	71° 10′
19° 00′	9.5126	37	0.4874	9.5370	41	0.4630	0.0243	4	9.9757	**71° 00′**
19° 10′	9.5163	36	0.4837	9.5411	41	0.4589	0.0248	4	9.9752	70° 50′
19° 20′	9.5199	36	0.4801	9.5451	40	0.4549	0.0252	4	9.9748	70° 40′
19° 30′	9.5235	36	0.4765	9.5491	40	0.4509	0.0257	4	9.9743	70° 30′
19° 40′	9.5270	35	0.4730	9.5531	40	0.4469	0.0261	5	9.9739	70° 20′
19° 50′	9.5306	35	0.4694	9.5571	40	0.4429	0.0266	5	9.9734	70° 10′
20° 00′	9.5341	35	0.4659	9.5611	39	0.4389	0.0270	5	9.9730	**70° 00′**
20° 10′	9.5375	34	0.4625	9.5650	39	0.4350	0.0275	5	9.9725	69° 50′
20° 20′	9.5409	34	0.4591	9.5689	39	0.4311	0.0279	5	9.9721	69° 40′
20° 30′	9.5443	34	0.4557	9.5727	39	0.4273	0.0284	5	9.9716	69° 30′
20° 40′	9.5477	33	0.4523	9.5766	38	0.4234	0.0289	5	9.9711	69° 20′
20° 50′	9.5510	33	0.4490	9.5804	38	0.4196	0.0294	5	9.9706	69° 10′
21° 00′	9.5543	33	0.4457	9.5842	38	0.4158	0.0298	5	9.9702	**69° 00′**
21° 10′	9.5576	33	0.4424	9.5879	38	0.4121	0.0303	5	9.9697	68° 50′
21° 20′	9.5609	32	0.4391	9.5917	37	0.4083	0.0308	5	9.9692	68° 40′
21° 30′	9.5641	32	0.4359	9.5954	37	0.4046	0.0313	5	9.9687	68° 30′
21° 40′	9.5673	32	0.4327	9.5991	37	0.4009	0.0318	5	9.9682	68° 20′
21° 50′	9.5704	32	0.4296	9.6028	37	0.3972	0.0323	5	9.9677	68° 10′
22° 00′	9.5736	31	0.4264	9.6064	36	0.3936	0.0328	5	9.9672	**68° 00′**
22° 10′	9.5767	31	0.4233	9.6100	36	0.3900	0.0333	5	9.9667	67° 50′
22° 20′	9.5798	31	0.4202	9.6136	36	0.3864	0.0339	5	9.9661	67° 40′
22° 30′	9.5828	30	0.4172	9.6172	36	0.3828	0.0344	5	9.9656	67 30′
22° 40′	9.5859	30	0.4141	9.6208	36	0.3792	0.0349	5	9.9651	67° 20′
22° 50′	9.5889	30	0.4111	9.6243	35	0.3757	0.0354	5	9.9646	67° 10′
23° 00′	9.5919	30	0.4081	9.6279	35	0.3721	0.0360	5	9.9640	**67° 00′**
23° 10′	9.5948	30	0.4052	9.6314	35	0.3686	0.0365	5	9.9635	66° 50′
23° 20′	9.5978	29	0.4022	9.6348	35	0.3652	0.0371	5	9.9629	66° 40′
23° 30′	9.6007	29	0.3993	9.6383	35	0.3617	0.0376	5	9.9624	66° 30′
23° 40′	9.6036	29	0.3964	9.6417	34	0.3583	0.0382	6	9.9618	66° 20′
23° 50′	9.6065	29	0.3935	9.6452	34	0.3548	0.0387	6	9.9613	66° 10′
24° 00′	9.6093	28	0.3907	9.6486	34	0.3514	0.0393	6	9.9607	**66° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

ϕ	l sin ϕ		l csc ϕ	l tan ϕ		l ctn ϕ	l sec ϕ		l cos ϕ	
24° 00′	9.6093	28	0.3907	9.6486	34	0.3514	0.0393	6	9.9607	**66° 00′**
24° 10′	9.6121	28	0.3879	9.6520	34	0.3480	0.0398	6	9.9602	65° 50′
24° 20′	9.6149	28	0.3851	9.6553	34	0.3447	0.0404	6	9.9596	65° 40′
24° 30′	9.6177	28	0.3823	9.6587	33	0.3413	0.0410	6	9.9590	65° 30′
24° 40′	9.6205	28	0.3795	9.6620	33	0.3380	0.0416	6	9.9584	65° 20′
24° 50′	9.6232	27	0.3768	9.6654	33	0.3346	0.0421	6	9.9579	65° 10′
25° 00′	9.6259	27	0.3741	9.6687	33	0.3313	0.0427	6	9.9573	**65° 00′**
25° 10′	9.6286	27	0.3714	9.6720	33	0.3280	0.0433	6	9.9567	64° 50′
25° 20′	9.6313	27	0.3687	9.6752	33	0.3248	0.0439	6	9.9561	64° 40′
25° 30′	9.6340	26	0.3660	9.6785	33	0.3215	0.0445	6	9.9555	64° 30′
25° 40′	9.6366	26	0.3634	9.6817	32	0.3183	0.0451	6	9.9549	64° 20′
25° 50′	9.6392	26	0.3608	9.6850	32	0.3150	0.0457	6	9.9543	64° 10′
26° 00′	9.6418	26	0.3582	9.6882	32	0.3118	0.0463	6	9.9537	**64° 00′**
26° 10′	9.6444	26	0.3556	9.6914	32	0.3086	0.0470	6	9.9530	63° 50′
26° 20′	9.6470	26	0.3530	9.6946	32	0.3054	0.0476	6	9.9524	63° 40′
26° 30′	9.6495	25	0.3505	9.6977	32	0.3023	0.0482	6	9.9518	63° 30′
26° 40′	9.6521	25	0.3479	9.7009	31	0.2991	0.0488	6	9.9512	63° 20′
26° 50′	9.6546	25	0.3454	9.7040	31	0.2960	0.0495	6	9.9505	63° 10′
27° 00′	9.6570	25	0.3430	9.7072	31	0.2928	0.0501	6	9.9499	**63° 00′**
27° 10′	9.6595	25	0.3405	9.7103	31	0.2897	0.0508	6	9.9492	62° 50′
27° 20′	9.6620	24	0.3380	9.7134	31	0.2866	0.0514	7	9.9486	62° 40′
27° 30′	9.6644	24	0.3356	9.7165	31	0.2835	0.0521	7	9.9479	62° 30′
27° 40′	9.6668	24	0.3332	9.7196	31	0.2804	0.0527	7	9.9473	62° 20′
27° 50′	9.6692	24	0.3308	9.7226	31	0.2774	0.0534	7	9.9466	62° 10′
28° 00′	9.6716	24	0.3284	9.7257	30	0.2743	0.0541	7	9.9459	**62° 00′**
28° 10′	9.6740	24	0.3260	9.7287	30	0.2713	0.0547	7	9.9453	61° 50′
28° 20′	9.6763	23	0.3237	9.7317	30	0.2683	0.0554	7	9.9446	61° 40′
28° 30′	9.6787	23	0.3213	9.7348	30	0.2652	0.0561	7	9.9439	61° 30′
28° 40′	9.6810	23	0.3190	9.7378	30	0.2622	0.0568	7	9.9432	61° 20′
28° 50′	9.6833	23	0.3167	9.7408	30	0.2592	0.0575	7	9.9425	61° 10′
29° 00′	9.6856	23	0.3144	9.7438	30	0.2562	0.0582	7	9.9418	**61° 00′**
29° 10′	9.6878	23	0.3122	9.7467	30	0.2533	0.0589	7	9.9411	60° 50′
29° 20′	9.6901	22	0.3099	9.7497	30	0.2503	0.0596	7	9.9404	60° 40′
29° 30′	9.6923	22	0.3077	9.7526	29	0.2474	0.0603	7	9.9397	60° 30′
29° 40′	9.6946	22	0.3054	9.7556	29	0.2444	0.0610	7	9.9390	60° 20′
29° 50′	9.6968	22	0.3032	9.7585	29	0.2415	0.0617	7	9.9383	60° 10′
30° 00′	9.6990	22	0.3010	9.7614	29	0.2386	0.0625	7	9.9375	**60° 00′**
30° 10′	9.7012	22	0.2988	9.7644	29	0.2356	0.0632	7	9.9368	59° 50′
30° 20′	9.7033	22	0.2967	9.7673	29	0.2327	0.0639	7	9.9361	59° 40′
30° 30′	9.7055	21	0.2945	9.7701	29	0.2299	0.0647	7	9.9353	59° 30′
30° 40′	9.7076	21	0.2924	9.7730	29	0.2270	0.0654	7	9.9346	59° 20′
30° 50′	9.7097	21	0.2903	9.7759	29	0.2241	0.0662	8	9.9338	59° 10′
31° 00′	9.7118	21	0.2882	9.7788	29	0.2212	0.0669	8	9.9331	**59° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

ϕ	l sin ϕ		l csc ϕ	l tan ϕ		l ctn ϕ	l sec ϕ		l cos ϕ	
31° 00′	9.7118	21	0.2882	9.7788	29	0.2212	0.0669	8	9.9331	**59° 00′**
31° 10′	9.7139	21	0.2861	9.7816	29	0.2184	0.0677	8	9.9323	58° 50′
31° 20′	9.7160	21	0.2840	9.7845	28	0.2155	0.0685	8	9.9315	58° 40′
31° 30′	9.7181	21	0.2819	9.7873	28	0.2127	0.0692	8	9.9308	58° 30′
31° 40′	9.7201	20	0.2799	9.7902	28	0.2098	0.0700	8	9.9300	58° 20′
31° 50′	9.7222	20	0.2778	9.7930	28	0.2070	0.0708	8	9.9292	58° 10′
32° 00′	9.7242	20	0.2758	9.7958	28	0.2042	0.0716	8	9.9284	**58° 00′**
32° 10′	9.7262	20	0.2738	9.7986	28	0.2014	0.0724	8	9.9276	57° 50′
32° 20′	9.7282	20	0.2718	9.8014	28	0.1986	0.0732	8	9.9268	57° 40′
32° 30′	9.7302	20	0.2698	9.8042	28	0.1958	0.0740	8	9.9260	57° 30′
32° 40′	9.7322	20	0.2678	9.8070	28	0.1930	0.0748	8	9.9252	57° 20′
32° 50′	9.7342	20	0.2658	9.8097	28	0.1903	0.0756	8	9.9244	57° 10′
33° 00′	9.7361	19	0.2639	9.8125	28	0.1875	0.0764	8	9.9236	**57° 00′**
33° 10′	9.7380	19	0.2620	9.8153	28	0.1847	0.0772	8	9.9228	56° 50′
33° 20′	9.7400	19	0.2600	9.8180	28	0.1820	0.0781	8	9.9219	56° 40′
33° 30′	9.7419	19	0.2581	9.8208	27	0.1792	0.0789	8	9.9211	56° 30′
33° 40′	9.7438	19	0.2562	9.8235	27	0.1765	0.0797	8	9.9203	56° 20′
33° 50′	9.7457	19	0.2543	9.8263	27	0.1737	0.0806	8	9.9194	56° 10′
34° 00′	9.7476	19	0.2524	9.8290	27	0.1710	0.0814	9	9.9186	**56° 00′**
34° 10′	9.7494	19	0.2506	9.8317	27	0.1683	0.0823	9	9.9177	55° 50′
34° 20′	9.7513	18	0.2487	9.8344	27	0.1656	0.0831	9	9.9169	55° 40′
34° 30′	9.7531	18	0.2469	9.8371	27	0.1629	0.0840	9	9.9160	55° 30′
34° 40′	9.7550	18	0.2450	9.8398	27	0.1602	0.0849	9	9.9151	55° 20′
34° 50′	9.7568	18	0.2432	9.8425	27	0.1575	0.0858	9	9.9142	55° 10′
35° 00′	9.7586	18	0.2414	9.8452	27	0.1548	0.0866	9	9.9134	**55° 00′**
35° 10′	9.7604	18	0.2396	9.8479	27	0.1521	0.0875	9	9.9125	54° 50′
35° 20′	9.7622	18	0.2378	9.8506	27	0.1494	0.0884	9	9.9116	54° 40′
35° 30′	9.7640	18	0.2360	9.8533	27	0.1467	0.0893	9	9.9107	54° 30′
35° 40′	9.7657	18	0.2343	9.8559	27	0.1441	0.0902	9	9.9098	54° 20′
35° 50′	9.7675	17	0.2325	9.8586	27	0.1414	0.0911	9	9.9089	54° 10′
36° 00′	9.7692	17	0.2308	9.8613	27	0.1387	0.0920	9	9.9080	**54° 00′**
36° 10′	9.7710	17	0.2290	9.8639	27	0.1361	0.0930	9	9.9070	53° 50′
36° 20′	9.7727	17	0.2273	9.8666	26	0.1334	0.0939	9	9.9061	53° 40′
36° 30′	9.7744	17	0.2256	9.8692	26	0.1308	0.0948	9	9.9052	53° 30′
36° 40′	9.7761	17	0.2239	9.8718	26	0.1282	0.0958	9	9.9042	53° 20′
36° 50′	9.7778	17	0.2222	9.8745	26	0.1255	0.0967	9	9.9033	53° 10′
37° 00′	9.7795	17	0.2205	9.8771	26	0.1229	0.0977	10	9.9023	**53° 00′**
37° 10′	9.7811	17	0.2189	9.8797	26	0.1203	0.0986	10	9.9014	52° 50′
37° 20′	9.7828	17	0.2172	9.8824	26	0.1176	0.0996	10	9.9004	52° 40′
37° 30′	9.7844	16	0.2156	9.8850	26	0.1150	0.1005	10	9.8995	52° 30′
37° 40′	9.7861	16	0.2139	9.8876	26	0.1124	0.1015	10	9.8985	52° 20′
37° 50′	9.7877	16	0.2123	9.8902	26	0.1098	0.1025	10	9.8975	52° 10′
38° 00′	9.7893	16	0.2107	9.8928	26	0.1072	0.1035	10	9.8965	**52° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

ϕ	l sin ϕ		l csc ϕ	l tan ϕ		l ctn ϕ	l sec ϕ		l cos ϕ	
38° 00′	9.7893	16	0.2107	9.8928	26	0.1072	0.1035	10	9.8965	**52° 00′**
38° 10′	9.7910	16	0.2090	9.8954	26	0.1046	0.1045	10	9.8955	51° 50′
38° 20′	9.7926	16	0.2074	9.8980	26	0.1020	0.1055	10	9.8945	51° 40′
38° 30′	9.7941	16	0.2059	9.9006	26	0.0994	0.1065	10	9.8935	51° 30′
38° 40′	9.7957	16	0.2043	9.9032	26	0.0968	0.1075	10	9.8925	51° 20′
38° 50′	9.7973	16	0.2027	9.9058	26	0.0942	0.1085	10	9.8915	51° 10′
39° 00′	9.7989	16	0.2011	9.9084	26	0.0916	0.1095	10	9.8905	**51° 00′**
39° 10′	9.8004	16	0.1996	9.9110	26	0.0890	0.1105	10	9.8895	50° 50′
39° 20′	9.8020	15	0.1980	9.9135	26	0.0865	0.1116	10	9.8884	50° 40′
39° 30′	9.8035	15	0.1965	9.9161	26	0.0839	0.1126	10	9.8874	50° 30′
39° 40′	9.8050	15	0.1950	9.9187	26	0.0813	0.1136	10	9.8864	50° 20′
39° 50′	9.8066	15	0.1934	9.9212	26	0.0788	0.1147	11	9.8853	50° 10′
40° 00′	9.8081	15	0.1919	9.9238	26	0.0762	0.1157	11	9.8843	**50° 00′**
40° 10′	9.8096	15	0.1904	9.9264	26	0.0736	0.1168	11	9.8832	49° 50′
40° 20′	9.8111	15	0.1889	9.9289	26	0.0711	0.1179	11	9.8821	49° 40′
40° 30′	9.8125	15	0.1875	9.9315	26	0.0685	0.1190	11	9.8810	49° 30′
40° 40′	9.8140	15	0.1860	9.9341	26	0.0659	0.1200	11	9.8800	49° 20′
40° 50′	9.8155	15	0.1845	9.9366	26	0.0634	0.1211	11	9.8789	49° 10′
41° 00′	9.8169	15	0.1831	9.9392	26	0.0608	0.1222	11	9.8778	**49° 00′**
41° 10′	9.8184	14	0.1816	9.9417	25	0.0583	0.1233	11	9.8767	48° 50′
41° 20′	9.8198	14	0.1802	9.9443	25	0.0557	0.1244	11	9.8756	48° 40′
41° 30′	9.8213	14	0.1787	9.9468	25	0.0532	0.1255	11	9.8745	48° 30′
41° 40′	9.8227	14	0.1773	9.9494	25	0.0506	0.1267	11	9.8733	48° 20′
41° 50′	9.8241	14	0.1759	9.9519	25	0.0481	0.1278	11	9.8722	48° 10′
42° 00′	9.8255	14	0.1745	9.9544	25	0.0456	0.1289	11	9.8711	**48° 00′**
42° 10′	9.8269	14	0.1731	9.9570	25	0.0430	0.1301	11	9.8699	47° 50′
42° 20′	9.8283	14	0.1717	9.9595	25	0.0405	0.1312	12	9.8688	47° 40′
42° 30′	9.8297	14	0.1703	9.9621	25	0.0379	0.1324	12	9.8676	47° 30′
42° 40′	9.8311	14	0.1689	9.9646	25	0.0354	0.1335	12	9.8665	47° 20′
42° 50′	9.8324	14	0.1676	9.9671	25	0.0329	0.1347	12	9.8653	47° 10′
43° 00′	9.8338	14	0.1662	9.9697	25	0.0303	0.1359	12	9.8641	**47° 00′**
43° 10′	9.8351	13	0.1649	9.9722	25	0.0278	0.1371	12	9.8629	46° 50′
43° 20′	9.8365	13	0.1635	9.9747	25	0.0253	0.1382	12	9.8618	46° 40′
43° 30′	9.8378	13	0.1622	9.9772	25	0.0228	0.1394	12	9.8606	46° 30′
43° 40′	9.8391	13	0.1609	9.9798	25	0.0202	0.1406	12	9.8594	46° 20′
43° 50′	9.8405	13	0.1595	9.9823	25	0.0177	0.1418	12	9.8582	46° 10′
44° 00′	9.8418	13	0.1582	9.9848	25	0.0152	0.1431	12	9.8569	**46° 00′**
44° 10′	9.8431	13	0.1569	9.9874	25	0.0126	0.1443	12	9.8557	45° 50′
44° 20′	9.8444	13	0.1556	9.9899	25	0.0101	0.1455	12	9.8545	45° 40′
44° 30′	9.8457	13	0.1543	9.9924	25	0.0076	0.1468	12	9.8532	45° 30′
44° 40′	9.8469	13	0.1531	9.9949	25	0.0051	0.1480	12	9.8520	45° 20′
44° 50′	9.8482	13	0.1518	9.9975	25	0.0025	0.1493	13	9.8507	45° 10′
45° 00′	9.8495	13	0.1505	0.0000	25	0.0000	0.1505	13	9.8495	**45° 00′**
	l cos θ		l sec θ	l ctn θ		l tan θ	l csc θ		l sin θ	θ

$\log u$	$\sin^{-1}u$		$\cos^{-1}u$	$\tan^{-1}u$		$\operatorname{ctn}^{-1}u$	$\log u$	$\sin^{-1}u$		$\cos^{-1}u$	$\tan^{-1}u$		$\operatorname{ctn}^{-1}u$
9.	°		°	°		°	**9.**	°		°	°		°
00	5.74	13	84.26	5.71	13	84.29	**50**	18.43	44	71.57	17.55	38	72.45
01	5.87	14	84.13	5.84	13	84.16	51	18.88	45	71.12	17.93	39	72.07
02	6.01	14	83.99	5.98	14	84.02	52	19.34	46	70.66	18.32	39	71.68
03	6.15	14	83.85	6.12	14	83.88	53	19.81	48	70.19	18.72	40	71.28
04	6.30	15	83.70	6.26	14	83.74	54	20.29	49	69.71	19.12	41	70.88
05	6.44	15	83.56	6.40	15	83.60	**55**	20.78	50	69.22	19.54	42	70.46
06	6.59	15	83.41	6.55	15	83.45	56	21.29	51	68.71	19.95	42	70.05
07	6.75	16	83.25	6.70	15	83.30	57	21.81	53	68.19	20.38	43	69.62
08	6.91	16	83.09	6.86	16	83.14	58	22.35	54	67.65	20.82	44	69.18
09	7.07	16	82.93	7.01	16	82.99	59	22.90	56	67.10	21.26	45	68.74
10	7.23	17	82.77	7.18	16	82.82	**60**	23.46	57	66.54	21.71	45	68.29
11	7.40	17	82.60	7.34	17	82.66	61	24.04	59	65.96	22.17	46	67.83
12	7.58	18	82.42	7.51	17	82.49	62	24.64	61	65.36	22.63	47	67.37
13	7.75	18	82.25	7.68	17	82.32	63	25.25	62	64.75	23.10	48	69.90
14	7.93	18	82.07	7.86	18	82.14	64	25.88	64	64.12	23.58	48	66.42
15	8.12	19	81.88	8.04	18	81.96	**65**	26.53	66	63.47	24.07	49	65.93
16	8.31	19	81.69	8.22	19	81.78	66	27.20	68	62.80	24.56	50	65.44
17	8.51	20	81.49	8.41	19	81.59	67	27.89	70	62.11	25.07	51	64.93
18	8.71	20	81.29	8.61	20	81.39	68	28.60	72	61.40	25.58	51	64.42
19	8.91	21	81.09	8.80	20	81.20	69	29.33	74	60.67	26.09	52	63.91
20	9.12	21	80.88	9.01	20	80.99	**70**	30.08	76	59.92	26.62	53	63.38
21	9.33	22	80.67	9.21	21	80.79	71	30.85	79	59.15	27.15	54	62.85
22	9.55	22	80.45	9.42	21	80.58	72	31.66	81	58.34	27.69	54	62.31
23	9.78	23	80.22	9.64	22	80.36	73	32.48	84	57.52	28.24	55	61.76
24	10.01	23	79.99	9.86	22	80.14	74	33.34	87	56.66	28.79	56	61.21
25	10.24	24	79.76	10.08	23	79.92	**75**	34.22	90	55.78	29.35	56	60.65
26	10.48	24	79.52	10.31	23	79.69	76	35.13		54.87	29.92	57	60.08
27	10.73	25	79.27	10.55	24	79.45	77	36.07		53.93	30.49	58	59.51
28	10.98	26	79.02	10.79	24	79.21	78	37.05		52.95	31.07	58	58.93
29	11.24	26	78.76	11.03	25	78.97	79	38.07		51.93	31.66	59	58.34
30	11.51	27	78.49	11.28	25	78.72	**80**	39.12		50.88	32.25	60	57.75
31	11.78	28	78.22	11.54	26	78.46	81	40.21		49.79	32.85	60	57.15
32	12.06	28	77.94	11.80	26	78.20	82	41.35		48.65	33.45	61	56.55
33	12.34	29	77.66	12.07	27	77.93	83	42.54		47.46	34.06	61	55.94
34	12.64	30	77.36	12.34	28	77.66	84	43.78		46.22	34.68	62	55.32
35	12.94	30	77.06	12.62	28	77.38	**85**	45.07		44.93	35.30	62	54.70
36	13.24	31	76.76	12.90	29	77.10	86	46.42		43.58	35.92	63	54.08
37	13.56	32	76.44	13.19	29	76.81	87	47.84		42.16	36.55	63	53.45
38	13.88	33	76.12	13.49	30	76.51	88	49.34		40.66	37.18	64	52.82
39	14.21	33	75.79	13.79	31	76.21	89	50.92		39.08	37.82	64	52.18
40	14.55	34	75.45	14.10	31	75.90	**90**	52.59		37.41	38.46	64	51.54
41	14.89	35	75.11	14.42	32	75.58	91	54.37		35.63	39.11	65	50.89
42	15.25	36	74.75	14.74	32	75.26	92	56.28		33.72	39.75	65	50.25
43	15.61	37	74.39	15.06	33	74.94	93	58.34		31.66	40.40	65	49.60
44	15.99	38	74.01	15.40	34	74.60	94	60.57		29.43	41.05	65	48.95
45	16.37	39	73.63	15.74	34	74.26	**95**	63.03		26.97	41.71	66	48.29
46	16.76	40	73.24	16.09	35	73.91	96	65.78		24.22	42.37	66	47.63
47	17.16	41	72.84	16.44	36	73.56	97	68.95		21.05	43.02	66	46.98
48	17.58	42	72.42	16.80	37	73.20	98	72.74		17.26	43.68	66	46.32
49	18.00	43	72.00	17.17	37	72.83	99	77.75		12.25	44.34	66	45.66
50	18.43	44	71.57	17.55	38	72.45	**00**	90.00		00.00	45.00	66	45.00

log u	$\sin^{-1} u$		$\cos^{-1} u$	log u	$\sin^{-1} u$		$\cos^{-1} u$	log u	$\sin^{-1} u$		$\cos^{-1} u$
9.	°		°	**9.**	°		°	**9.**	°		°
750	34.22	9	55.78	**800**	39.12	11	50.88	**850**	45.07	13	44.93
751	34.31	9	55.69	801	39.23	11	50.77	851	45.20	13	44.80
752	34.40	9	55.60	802	39.34	11	50.66	852	45.33	13	44.67
753	34.49	9	55.51	803	39.44	11	50.56	853	45.47	13	44.53
754	34.58	9	55.42	804	39.55	11	50.45	854	45.60	13	44.40
755	34.67	9	55.33	**805**	39.66	11	50.34	**855**	45.74	14	44.26
756	34.76	9	55.24	806	39.77	11	50.23	856	45.87	14	44.13
757	34.85	9	55.15	807	39.88	11	50.12	857	46.01	14	43.99
758	34.95	9	55.05	808	39.99	11	50.01	858	46.15	14	43.85
759	35.04	9	54.96	809	40.10	11	49.90	859	46.28	14	43.72
760	35.13	9	54.87	**810**	40.21	11	49.79	**860**	46.42	14	43.58
761	35.22	9	54.78	811	40.33	11	49.67	861	46.56	14	43.44
762	35.32	9	54.68	812	40.44	11	49.56	862	46.70	14	43.30
763	35.41	9	54.59	813	40.55	11	49.45	863	46.84	14	43.16
764	35.50	9	54.50	814	40.66	11	49.34	864	46.98	14	43.02
765	35.60	9	54.40	**815**	40.78	11	49.22	**865**	47.12	14	42.88
766	35.69	9	54.31	816	40.89	11	49.11	866	47.27	14	42.73
767	35.79	10	54.21	817	41.01	11	48.99	867	47.41	14	42.59
768	35.88	10	54.12	818	41.12	12	48.88	868	47.55	14	42.45
769	35.98	10	54.02	819	41.24	12	48.76	869	47.70	14	42.30
770	36.07	10	53.93	**820**	41.35	12	48.65	**870**	47.84	15	42.16
771	36.17	10	53.83	821	41.47	12	48.53	871	47.99	15	42.01
772	36.27	10	53.73	822	41.59	12	48.41	872	48.14	15	41.86
773	36.36	10	53.64	823	41.70	12	48.30	873	48.28	15	41.72
774	36.46	10	53.54	824	41.82	12	48.18	874	48.43	15	41.57
775	36.56	10	53.44	**825**	41.94	12	48.06	**875**	48.58	15	41.42
776	36.66	10	53.34	826	42.06	12	47.94	876	48.73	15	41.27
777	36.76	10	53.24	827	42.18	12	47.82	877	48.88	15	41.12
778	36.85	10	53.15	828	42.30	12	47.70	878	49.03	15	40.97
779	36.95	10	53.05	829	42.42	12	47.58	879	49.19	15	40.81
780	37.05	10	52.95	**830**	42.54	12	47.46	**880**	49.34	15	40.66
781	37.15	10	52.85	831	42.66	12	47.34	881	49.49	15	40.51
782	37.25	10	52.75	832	42.78	12	47.22	882	49.65	16	40.35
783	37.35	10	52.65	833	42.90	12	47.10	883	49.80	16	40.20
784	37.45	10	52.55	834	43.03	12	46.97	884	49.96	16	40.04
785	37.56	10	52.44	**835**	43.15	12	46.85	**885**	50.12	16	39.88
786	37.66	10	52.34	836	43.27	12	46.73	886	50.28	16	39.72
787	37.76	10	52.24	837	43.40	12	46.60	887	50.44	16	39.56
788	37.86	10	52.14	838	43.52	13	46.48	888	50.60	16	39.40
789	37.96	10	52.04	839	43.65	13	46.35	889	50.76	16	39.24
790	38.07	10	51.93	**840**	43.78	13	46.22	**890**	50.92	16	39.08
791	38.17	10	51.83	841	43.90	13	46.10	891	51.08	16	38.92
792	38.28	10	51.72	842	44.03	13	45.97	892	51.25	16	38.75
793	38.38	10	51.62	843	44.16	13	45.84	893	51.41	17	38.59
794	38.48	10	51.52	844	44.29	13	45.71	894	51.58	17	38.42
795	38.59	11	51.41	**845**	44.41	13	45.59	**895**	51.74	17	38.26
796	38.69	11	51.31	846	44.54	13	45.46	896	51.91	17	38.09
797	38.80	11	51.20	847	44.67	13	45.33	897	52.08	17	37.92
798	38.91	11	51.09	848	44.80	13	45.20	898	52.25	17	37.75
799	39.01	11	50.99	849	44.94	13	45.06	899	52.42	17	37.58
800	39.12	11	50.88	**850**	45.07	13	44.93	**900**	52.59	17	37.41

log u	sin⁻¹u		cos⁻¹u	log u	sin⁻¹u		cos⁻¹u	log u	sin⁻¹u	log u	sin⁻¹u
9.	°		°	9.	°		°	9.	°	9.	°
900	52.59	17	37.41	**950**	63.03	26	26.97	**9900**	77.75	**9950**	81.32
901	52.76	17	37.24	951	63.29	26	26.71	9901	77.81	9951	81.41
902	52.94	17	37.06	952	63.56	27	26.44	9902	77.87	9952	81.50
903	53.11	18	36.89	953	63.82	27	26.18	9903	77.94	9953	81.59
904	53.29	18	36.71	954	64.09	27	25.91	9904	78.00	9954	81.68
905	53.47	18	36.53	**955**	64.37	27	25.63	**9905**	78.06	**9955**	81.77
906	53.65	18	36.35	956	64.64	28	25.36	9906	78.12	9956	81.86
907	53.83	18	36.17	957	64.92	28	25.08	9907	78.18	9957	81.95
908	54.01	18	35.99	958	65.21	29	24.79	9908	78.25	9958	82.04
909	54.19	18	35.81	959	65.49	29	24.51	9909	78.31	9959	82.14
910	54.37	18	35.63	**960**	65.78	29	24.22	**9910**	78.38	**9960**	82.24
911	54.56	19	35.44	961	66.08	30	23.92	9911	78.44	9961	82.33
912	54.74	19	35.26	962	66.38	30	23.62	9912	78.50	9962	82.43
913	54.93	19	35.07	963	66.68	31	23.32	9913	78.57	9963	82.5
914	55.12	19	34.88	964	66.99	31	23.01	9914	78.64	9964	82.6
915	55.31	19	34.69	**965**	67.30	32	22.70	**9915**	78.70	**9965**	82.7
916	55.50	19	34.50	966	67.62	32	22.38	9916	78.77	9966	82.8
917	55.69	19	34.31	967	67.95	33	22.05	9917	78.83	9967	82.9
918	55.89	19	34.11	968	68.27	33	21.73	9918	78.90	9968	83.1
919	56.08	20	33.92	969	68.61	34	21.39	9919	78.97	9969	83.2
920	56.28	20	33.72	**970**	68.95	34	21.05	**9920**	79.04	**9970**	83.3
921	56.48	20	33.52	971	69.29	35	20.71	9921	79.10	9971	83.4
922	56.68	20	33.32	972	69.65	36	20.35	9922	79.17	9972	83.5
923	56.88	20	33.12	973	70.01	36	19.99	9923	79.24	9973	83.6
924	57.08	20	32.92	974	70.37	37	19.63	9924	79.31	9974	83.7
925	57.29	21	32.71	**975**	70.75	38	19.25	**9925**	79.38	**9975**	83.9
926	57.49	21	32.51	976	71.13	39	18.87	9926	79.45	9976	84.0
927	57.70	21	32.30	977	71.52	39	18.48	9927	79.52	9977	84.1
928	57.91	21	32.09	978	71.92	40	18.08	9928	79.60	9978	84.2
929	58.12	21	31.88	979	72.33	41	17.67	9929	79.67	9979	84.4
930	58.34	21	31.66	**980**	72.74	42	17.26	**9930**	79.74	**9980**	84.5
931	58.55	22	31.45	981	73.18	44	16.82	9931	79.81	9981	84.6
932	58.77	22	31.23	982	73.62	45	16.38	9932	79.89	9982	84.8
933	58.99	22	31.01	983	74.07	46	15.93	9933	79.96	9983	84.9
934	59.21	22	30.79	984	74.54	48	15.46	9934	80.04	9984	85.1
935	59.43	22	30.57	**985**	75.03	49	14.97	**9935**	80.11	**9985**	85.2
936	59.65	23	30.35	986	75.53	51	14.47	9936	80.19	9986	85.4
937	59.88	23	30.12	987	76.05	53	13.95	9937	80.26	9987	85.6
938	60.11	23	29.89	988	76.59	55	13.41	9938	80.34	9988	85.7
939	60.34	23	29.66	989	77.16	58	12.84	9939	80.42	9989	85.9
940	60.57	23	29.43	**990**	77.75	61	12.25	**9940**	80.50	**9990**	86.1
941	60.81	24	29.19	991	78.38		11.62	9941	80.58	9991	86.3
942	61.04	24	28.96	992	79.04		10.96	9942	80.66	9992	86.5
943	61.28	24	28.72	993	79.74		10.26	9943	80.74	9993	86.7
944	61.52	24	28.48	994	80.50		9.50	9944	80.82	9994	87.0
945	61.77	25	28.23	**995**	81.32		8.68	**9945**	80.90	**9995**	87.3
946	62.02	25	27.98	996	82.24		7.76	9946	80.98	9996	87.5
947	62.27	25	27.73	997	83.3		6.7	9947	81.07	9997	87.9
948	62.52	25	27.48	998	84.5		5.5	9948	81.15	9998	88.3
949	62.77	26	27.23	999	86.1		3.9	9949	81.24	9999	88.8
950	63.03	26	26.97	**000**	90.		0.	**9950**	81.32	**0000**	90.

x	gd x		x	gd x		x	l Sh x		l Ch x		l Th x	
	°			°			0.		0.		9.	
0.00	0.0000	5730	**0.50**	27.524	508	**1.00**	0701	57	1884	33	8817	24
0.01	0.5729	5729	0.51	28.031	506	1.01	0758	57	1917	33	8840	23
0.02	1.1458	5728	0.52	28.535	503	1.02	0815	56	1950	33	8864	23
0.03	1.7186	5727	0.53	29.037	501	1.03	0871	56	1984	34	8887	23
0.04	2.2912	5725	0.54	29.537	498	1.04	0927	56	2018	34	8909	22
0.05	2.8636	5722	**0.55**	30.034	496	**1.05**	0982	56	2051	34	8931	22
0.06	3.4357	5719	0.56	30.529	494	1.06	1038	55	2086	34	8952	21
0.07	4.0074	5716	0.57	31.021	491	1.07	1093	55	2120	34	8973	21
0.08	4.5788	5711	0.58	31.511	488	1.08	1148	55	2154	34	8994	20
0.09	5.1497	5706	0.59	31.998	486	1.09	1203	54	2189	35	9014	20
0.10	5.720	570	**0.60**	32.483	483	**1.10**	1257	54	2223	35	9034	19
0.11	6.290	570	0.61	32.965	481	1.11	1311	54	2258	35	9053	19
0.12	6.859	569	0.62	33.444	478	1.12	1365	54	2293	35	9072	19
0.13	7.428	568	0.63	33.921	475	1.13	1419	54	2328	35	9090	18
0.14	7.995	567	0.64	34.395	473	1.14	1472	53	2364	35	9108	18
0.15	8.562	567	**0.65**	34.867	470	**1.15**	1525	53	2399	36	9126	18
0.16	9.128	566	0.66	35.336	467	1.16	1578	53	2435	36	9144	17
0.17	9.694	565	0.67	35.802	465	1.17	1631	53	2470	36	9161	17
0.18	10.258	564	0.68	36.265	462	1.18	1684	52	2506	36	9177	17
0.19	10.821	563	0.69	36.726	459	1.19	1736	52	2542	36	9194	16
0.20	11.384	562	**0.70**	37.183	456	**1.20**	1788	52	2578	36	9210	16
0.21	11.945	561	0.71	37.638	454	1.21	1840	52	2615	36	9226	16
0.22	12.505	559	0.72	38.091	451	1.22	1892	52	2651	36	9241	15
0.23	13.063	558	0.73	38.540	448	1.23	1944	52	2688	37	9256	15
0.24	13.621	557	0.74	38.987	445	1.24	1995	51	2724	37	9271	15
0.25	14.177	556	**0.75**	39.431	443	**1.25**	2046	51	2761	37	9285	14
0.26	14.732	554	0.76	39.872	440	1.26	2098	51	2798	37	9300	14
0.27	15.285	553	0.77	40.310	437	1.27	2148	51	2835	37	9314	14
0.28	15.837	551	0.78	40.746	434	1.28	2199	51	2872	37	9327	14
0.29	16.388	550	0.79	41.179	431	1.29	2250	51	2909	37	9341	13
0.30	16.937	548	**0.80**	41.608	428	**1.30**	2300	50	2947	37	9354	13
0.31	17.484	546	0.81	42.035	426	1.31	2351	50	2984	38	9367	13
0.32	18.030	545	0.82	42.460	423	1.32	2401	50	3022	38	9379	12
0.33	18.573	543	0.83	42.881	420	1.33	2451	50	3059	38	9391	12
0.34	19.116	541	0.84	43.299	417	1.34	2501	50	3097	38	9404	12
0.35	19.656	540	**0.85**	43.715	414	**1.35**	2551	50	3135	38	9415	12
0.36	20.195	538	0.86	44.128	411	1.36	2600	50	3173	38	9427	11
0.37	20.732	536	0.87	44.537	408	1.37	2650	49	3211	38	9438	11
0.38	21.267	534	0.88	44.944	406	1.38	2699	49	3249	38	9450	11
0.39	21.800	532	0.89	45.348	403	1.39	2748	49	3288	38	9460	11
0.40	22.331	530	**0.90**	45.750	400	**1.40**	2797	49	3326	38	9471	11
0.41	22.859	528	0.91	46.148	397	1.41	2846	49	3365	39	9482	10
0.42	23.386	526	0.92	46.544	394	1.42	2895	49	3403	39	9492	10
0.43	23.911	524	0.93	46.936	391	1.43	2944	49	3442	39	9502	10
0.44	24.434	522	0.94	47.326	388	1.44	2993	49	3481	39	9512	10
0.45	24.955	519	**0.95**	47.713	386	**1.45**	3041	48	3520	39	9522	10
0.46	25.473	517	0.96	48.097	383	1.46	3090	48	3559	39	9531	9
0.47	25.989	515	0.97	48.478	380	1.47	3138	48	3598	39	9540	9
0.48	26.503	513	0.98	48.857	377	1.48	3186	48	3637	39	9549	9
0.49	27.015	510	0.99	49.232	374	1.49	3234	48	3676	39	9558	9
0.50	27.524	508	**1.00**	49.605	371	**1.50**	3282	48	3715	39	9567	9

x	l Sh x	l Ch x	l Th x	x	l Sh x	l Ch x	l Th x
	0.	0.	9.		0.	0.	9.
1.50	3282 48	3715 39	9567 9	**2.00**	5595 45	5754 42	9841 3
1.51	3330 48	3754 39	9576 8	2.01	5640 45	5796 42	9844 3
1.52	3378 48	3794 39	9584 8	2.02	5685 45	5838 42	9847 3
1.53	3426 48	3833 40	9592 8	2.03	5730 45	5880 42	9850 3
1.54	3474 48	3873 40	9601 8	2.04	5775 45	5922 42	9853 3
1.55	3521 48	3913 40	9608 8	**2.05**	5820 45	5964 42	9856 3
1.56	3569 47	3952 40	9616 8	2.06	5865 45	6006 42	9859 3
1.57	3616 47	3992 40	9624 8	2.07	5910 45	6048 42	9862 3
1.58	3663 47	4032 40	9631 7	2.08	5955 45	6090 42	9864 3
1.59	3711 47	4072 40	9639 7	2.09	6000 45	6132 42	9867 3
1.60	3758 47	4112 40	9646 7	**2.10**	6044 45	6175 42	9870 3
1.61	3805 47	4152 40	9653 7	2.11	6089 45	6217 42	9872 3
1.62	3852 47	4192 40	9660 7	2.12	6134 45	6259 42	9875 3
1.63	3899 47	4232 40	9666 7	2.13	6178 45	6301 42	9877 2
1.64	3946 47	4273 40	9673 7	2.14	6223 45	6343 42	9880 2
1.65	3992 47	4313 40	9679 6	**2.15**	6268 45	6386 42	9882 2
1.66	4039 47	4353 40	9686 6	2.16	6312 45	6428 42	9884 2
1.67	4086 47	4394 40	9692 6	2.17	6357 45	6470 42	9887 2
1.68	4132 47	4434 41	9698 6	2.18	6401 45	6512 42	9889 2
1.69	4179 46	4475 41	9704 6	2.19	6446 45	6555 42	9891 2
1.70	4225 46	4515 41	9710 6	**2.20**	6491 45	6597 42	9893 2
1.71	4272 46	4556 41	9716 6	2.21	6535 44	6640 42	9895 2
1.72	4318 46	4597 41	9721 6	2.22	6580 44	6682 42	9898 2
1.73	4364 46	4637 41	9727 5	2.23	6624 44	6724 42	9900 2
1.74	4411 46	4678 41	9732 5	2.24	6668 44	6767 42	9902 2
1.75	4457 46	4719 41	9738 5	**2.25**	6713 44	6809 42	9904 2
1.76	4503 46	4760 41	9743 5	2.26	6757 44	6852 42	9905 2
1.77	4549 46	4801 41	9748 5	2.27	6802 44	6894 43	9907 2
1.78	4595 46	4842 41	9753 5	2.28	6846 44	6937 43	9909 2
1.79	4641 46	4883 41	9758 5	2.29	6890 44	6979 43	9911 2
1.80	4687 46	4924 41	9763 5	**2.30**	6935 44	7022 43	9913 2
1.81	4733 46	4965 41	9767 5	2.31	6979 44	7064 43	9914 2
1.82	4778 46	5006 41	9772 5	2.32	7023 44	7107 43	9916 2
1.83	4824 46	5048 41	9776 4	2.33	7067 44	7150 43	9918 2
1.84	4870 46	5089 41	9781 4	2.34	7112 44	7192 43	9919 2
1.85	4915 46	5130 41	9785 4	**2.35**	7156 44	7235 43	9921 2
1.86	4961 46	5172 41	9789 4	2.36	7200 44	7278 43	9923 2
1.87	5007 46	5213 41	9794 4	2.37	7244 44	7320 43	9924 2
1.88	5052 46	5254 41	9798 4	2.38	7289 44	7363 43	9926 1
1.89	5098 45	5296 41	9802 4	2.39	7333 44	7406 43	9927 1
1.90	5143 45	5337 42	9806 4	**2.40**	7377 44	7448 43	9929 1
1.91	5188 45	5379 42	9810 4	2.41	7421 44	7491 43	9930 1
1.92	5234 45	5421 42	9813 4	2.42	7465 44	7534 43	9931 1
1.93	5279 45	5462 42	9817 4	2,43	7509 44	7577 43	9933 1
1.94	5324 45	5504 42	9821 4	2.44	7553 44	7619 43	9934 1
1.95	5370 45	5545 42	9824 4	**2.45**	7597 44	7662 43	9935 1
1.96	5415 45	5587 42	9828 3	2.46	7642 44	7705 43	9937 1
1.97	5460 45	5629 42	9831 3	2.47	7686 44	7748 43	9938 1
1.98	5505 45	5671 42	9834 3	2.48	7730 44	7791 43	9939 1
1.99	5550 45	5713 42	9838 3	2.49	7774 44	7833 43	9940 1
2.00	5595 45	5754 42	9841 3	**2.50**	7818 44	7876 43	9941 1

x	l Sh x	l Ch x	l Th x
	0.	**0.**	**9.**
2.50	7818 44	7876 43	9941
2.51	7862 44	7919 43	9943
2.52	7906 44	7962 43	9944
2.53	7950 44	8005 43	9945
2.54	7994 44	8048 43	9946
2.55	8038 44	8091 43	9947
2.56	8082 44	8134 43	9948
2.57	8126 44	8176 43	9949
2.58	8169 44	8219 43	9950
2.59	8213 44	8262 43	9951
2.60	8257 44	8305 43	9952
2.61	8301 44	8348 43	9953
2.62	8345 44	8391 43	9954
2.63	8389 44	8434 43	9955
2.64	8433 44	8477 43	9956
2.65	8477 44	8520 43	9957
2.66	8521 44	8563 43	9958
2.67	8564 44	8606 43	9958
2.68	8608 44	8649 43	9959
2.69	8652 44	8692 43	9960
2.70	8696 44	8735 43	9961
2.71	8740 44	8778 43	9962
2.72	8784 44	8821 43	9962
2.73	8827 44	8864 43	9963
2.74	8871 44	8907 43	9964
2.75	8915 44	8951 43	9965
2.76	8959 44	8994 43	9965
2.77	9003 44	9037 43	9966
2.78	9046 44	9080 43	9967
2.79	9090 44	9123 43	9967
2.80	9134 44	9166 43	9968
2.81	9178 44	9209 43	9969
2.82	9221 44	9252 43	9969
2.83	9265 44	9295 43	9970
2.84	9309 44	9338 43	9970
2.85	9353 44	9382 43	9971
2.86	9396 44	9425 43	9972
2.87	9440 44	9468 43	9972
2.88	9484 44	9511 43	9973
2.89	9527 44	9554 43	9973
2.90	9571 44	9597 43	9974
2.91	9615 44	9641 43	9974
2.92	9658 44	9684 43	9975
2.93	9702 44	9727 43	9975
2.94	9746 44	9770 43	9976
2.95	9789 44	9813 43	9976
2.96	9833 44	9856 43	9977
2.97	9877 44	9900 43	9977
2.98	9920 44	9943 43	9978
2.99	9964 44	9986 43	9978
3.00	1.0008 44	1.0029 43	9978

x	l Sh x	l Ch x	l Th x
3.0	1.0008 436	1.0029 432	9.9978
3.1	1.0444 436	1.0462 433	9.9982
3.2	1.0880 436	1.0894 433	9.9986
3.3	1.1316 435	1.1327 433	9.9988
3.4	1.1751 435	1.1761 433	9.9990
3.5	1.2186 435	1.2194 434	9.9992
3.6	1.2621 435	1.2628 434	9.9994
3.7	1.3056 435	1.3061 434	9.9995
3.8	1.3491 435	1.3495 434	9.9996
3.9	1.3925 435	1.3929 434	9.9996
4.0	1.4360 435	1.4363 434	9.9997
4.1	1.4795 435	1.4797 434	9.9998
4.2	1.5229 434	1.5231 434	9.9998
4.3	1.5664 434	1.5665 434	9.9998
4.4	1.6098 434	1.6099 434	9.9999
4.5	1.6532 434	1.6533 434	9.9999
4.6	1.6967 434	1.6968 434	9.9999
4.7	1.7401 434	1.7402 434	9.9999
4.8	1.7836 434	1.7836 434	9.9999
4.9	1.8270 434	1.8270 434	0.0000
5.0	1.8704 434	1.8705 434	0.0000
5.1	1.9139 434	1.9139 434	0.0000
5.2	1.9573 434	1.9573 434	0.0000
5.3	2.0007 434	2.0007 434	0.0000
5.4	2.0442 434	2.0442 434	0.0000
5.5	2.0876 434	2.0876 434	0.0000
5.6	2.1310 434	2.1310 434	0.0000
5.7	2.1744 434	2.1745 434	0.0000
5.8	2.2179 434	2.2179 434	0.0000
5.9	2.2613 434	2.2613 434	0.0000
6.0	2.3047 4343	2.3047 4343	0.0000
7.0	2.7390 4343	2.7390 4343	0.0000
8.0	3.1733 4343	3.1733 4343	0.0000
9.0	3.6076 4343	3.6076 4343	0.0000
10.0	4.0419 4343	4.0419 4343	0.0000

For higher values:

$\log \mathrm{Sh}\, x = \log \mathrm{Ch}\, x = x\mu - 0.301030$;

$\mathrm{Sh}^{-1} u = \mathrm{Ch}^{-1} u = (\log u + 0.3010)\, \mu^{-1}$.

n	$n\mu$	$n\mu^{-1}$	n
1	0.434294	2.302585	1
2	0.868589	4.605170	2
3	1.302883	6.907755	3
4	1.737178	9.210340	4
5	2.171472	11.512925	5
6	2.605767	13.815511	6
7	3.040061	16.118096	7
8	3.474356	18.420681	8
9	3.908650	20.723266	9
10	4.342945	23.025851	10

φ	sin φ	cos φ	
0°	.000000	1.0000	90°
10′	.002909	1.0000	50′
20′	.005818	1.0000	40′
30′	.008727	1.0000	30′
40′	.011635	0.9999	20′
50′	.014544	0.9999	10′
1°	.017452	0.9998	89°
10′	.02036	0.9998	50′
20′	.02327	0.9997	40′
30′	.02618	0.9997	30′
40′	.02908	0.9996	20′
50′	.03199	0.9995	10′
2°	.03490	0.9994	88°
10′	.03781	0.9993	50′
20′	.04071	0.9992	40′
30′	.04362	0.9990	30′
40′	.04653	0.9989	20′
50′	.04943	0.9988	10′
3°	.05234	0.9986	87°
10′	.05524	0.9985	50′
20′	.05814	0.9983	40′
30′	.06105	0.9981	30′
40′	.06395	0.9980	20′
50′	.06685	0.9978	10′
4°	.06976	0.9976	86°
10′	.07266	0.9974	50′
20′	.07556	0.9971	40′
30′	.07846	0.9969	30′
40′	.08136	0.9967	20′
50′	.08426	0.9964	10′
5°	.08716	0.9962	85°
10′	.09005	0.9959	50′
20′	.09295	0.9957	40′
30′	.09585	0.9954	30′
40′	.09874	0.9951	20′
50′	.10164	0.9948	10′
6°	.10453	0.9945	84°
10′	.1074	0.9942	50′
20′	.1103	0.9939	40′
30′	.1132	0.9936	30′
40′	.1161	0.9932	20′
50′	.1190	0.9929	10′
7°	.1219	0.9925	83°
10′	.1248	0.9922	50′
20′	.1276	0.9918	40′
30′	.1305	0.9914	30′
			82°
	cos θ	sin θ	θ

φ	sin φ	cos φ	
7°			
30′	.1305	.9914	30′
40′	.1334	.9911	20′
50′	.1363	.9907	10′
8°	.1392	.9903	82°
10′	.1421	.9899	50′
20′	.1449	.9894	40′
30′	.1478	.9890	30′
40′	.1507	.9886	20′
50′	.1536	.9881	10′
9°	.1564	.9877	81°
10′	.1593	.9872	50′
20′	.1622	.9868	40′
30′	.1650	.9863	30′
40′	.1679	.9858	20′
50′	.1708	.9853	10′
10°	.1736	.9848	80°
10′	.1765	.9843	50′
20′	.1794	.9838	40′
30′	.1822	.9833	30′
40′	.1851	.9827	20′
50′	.1880	.9822	10′
11°	.1908	.9816	79°
10′	.1937	.9811	50′
20′	.1965	.9805	40′
30′	.1994	.9799	30′
40′	.2022	.9793	20′
50′	.2051	.9787	10′
12°	.2079	.9781	78°
10′	.2108	.9775	50′
20′	.2136	.9769	40′
30′	.2164	.9763	30′
40′	.2193	.9757	20′
50′	.2221	.9750	10′
13°	.2250	.9744	77°
10′	.2278	.9737	50′
20′	.2306	.9730	40′
30′	.2334	.9724	30′
40′	.2363	.9717	20′
50′	.2391	.9710	10′
14°	.2419	.9703	76°
10′	.2447	.9696	50′
20′	.2476	.9689	40′
30′	.2504	.9681	30′
40′	.2532	.9674	20′
50′	.2560	.9667	10′
15°	.2588	.9659	75°
	cos θ	sin θ	θ

φ	sin φ	cos φ	
15°	.2588	.9659	75°
10′	.2616	.9652	50′
20′	.2644	.9644	40′
30′	.2672	.9636	30′
40′	.2700	.9628	20′
50′	.2728	.9621	10′
16°	.2756	.9613	74°
10′	.2784	.9605	50′
20′	.2812	.9596	40′
30′	.2840	.9588	30′
40′	.2868	.9580	20′
50′	.2896	.9572	10′
17°	.2924	.9563	73°
10′	.2952	.9555	50′
20′	.2979	.9546	40′
30′	.3007	.9537	30′
40′	.3035	.9528	20′
50′	.3062	.9520	10′
18°	.3090	.9511	72°
10′	.3118	.9502	50′
20′	.3145	.9492	40′
30′	.3173	.9483	30′
40′	.3201	.9474	20′
50′	.3228	.9465	10′
19°	.3256	.9455	71°
10′	.3283	.9446	50′
20′	.3311	.9436	40′
30′	.3338	.9426	30′
40′	.3365	.9417	20′
50′	.3393	.9407	10′
20°	.3420	.9397	70°
10′	.3448	.9387	50′
20′	.3475	.9377	40′
30′	.3502	.9367	30′
40′	.3529	.9356	20′
50′	.3557	.9346	10′
21°	.3584	.9336	69°
10′	.3611	.9325	50′
20′	.3638	.9315	40′
30′	.3665	.9304	30′
40′	.3692	.9293	20′
50′	.3719	.9283	10′
22°	.3746	.9272	68°
10′	.3773	.9261	50′
20′	.3800	.9250	40′
30′	.3827	.9239	30′
			67°
	cos θ	sin θ	θ

φ	sin φ	cos φ	
22°			
30′	.3827	.9239	30′
40′	.3854	.9228	20′
50′	.3881	.9216	10′
23°	.3907	.9205	**67°**
10′	.3934	.9194	50′
20′	.3961	.9182	40′
30′	.3987	.9171	30′
40′	.4014	.9159	20′
50′	.4041	.9147	10′
24°	.4067	.9135	**66°**
10′	.4094	.9124	50′
20′	.4120	.9112	40′
30′	.4147	.9100	30′
40′	.4173	.9088	20′
50′	.4200	.9075	10′
25°	.4226	.9063	**65°**
10′	.4253	.9051	50′
20′	.4279	.9038	40′
30′	.4305	.9026	30′
40′	.4331	.9013	20′
50′	.4358	.9001	10′
26°	.4384	.8988	**64°**
10′	.4410	.8975	50′
20′	.4436	.8962	40′
30′	.4462	.8949	30′
40′	.4488	.8936	20′
50′	.4514	.8923	10′
27°	.4540	.8910	**63°**
10′	.4566	.8897	50′
20′	.4592	.8884	40′
30′	.4617	.8870	30′
40′	.4643	.8857	20′
50′	.4669	.8843	10′
28°	.4695	.8829	**62°**
10′	.4720	.8816	50′
20′	.4746	.8802	40′
30′	.4772	.8788	30′
40′	.4797	.8774	20′
50′	.4823	.8760	10′
29°	.4848	.8746	**61°**
10′	.4874	.8732	50′
20′	.4899	.8718	40′
30′	.4924	.8704	30′
40′	.4950	.8689	20′
50′	.4975	.8675	10′
30°	.5000	.8660	**60°**
	cos θ	sin θ	θ

φ	sin φ	cos φ	
30°	.5000	.8660	**60°**
10′	.5025	.8646	50′
20′	.5050	.8631	40′
30′	.5075	.8616	30′
40′	.5100	.8601	20′
50′	.5125	.8587	10′
31°	.5150	.8572	**59°**
10′	.5175	.8557	50′
20′	.5200	.8542	40′
30′	.5225	.8526	30′
40′	.5250	.8511	20′
50′	.5275	.8496	10′
32°	.5299	.8480	**58°**
10′	.5324	.8465	50′
20′	.5348	.8450	40′
30′	.5373	.8434	30′
40′	.5398	.8418	20′
50′	.5422	.8403	10′
33°	.5446	.8387	**57°**
10′	.5471	.8371	50′
20′	.5495	.8355	40′
30′	.5519	.8339	30′
40′	.5544	.8323	20′
50′	.5568	.8307	10′
34°	.5592	.8290	**56°**
10′	.5616	.8274	50′
20′	.5640	.8258	40′
30′	.5664	.8241	30′
40′	.5688	.8225	20′
50′	.5712	.8208	10′
35°	.5736	.8192	**55°**
10′	.5760	.8175	50′
20′	.5783	.8158	40′
30′	.5807	.8141	30′
40′	.5831	.8124	20′
50′	.5854	.8107	10′
36°	.5878	.8090	**54°**
10′	.5901	.8073	50′
20′	.5925	.8056	40′
30′	.5948	.8039	30′
40′	.5972	.8021	20′
50′	.5995	.8004	10′
37°	.6018	.7986	**53°**
10′	.6041	.7969	50′
20′	.6065	.7951	40′
30′	.6088	.7934	30′
			52°
	cos θ	sin θ	θ

φ	sin φ	cos φ	
37°			
30′	.6088	.7934	30′
40′	.6111	.7916	20′
50′	.6134	.7898	10′
38°	.6157	.7880	**52°**
10′	.6180	.7862	50′
20′	.6202	.7844	40′
30′	.6225	.7826	30′
40′	.6248	.7808	20′
50′	.6271	.7790	10′
39°	.6293	.7771	**51°**
10′	.6316	.7753	50′
20′	.6338	.7735	40′
30′	.6361	.7716	30′
40′	.6383	.7698	20′
50′	.6406	.7679	10′
40°	.6428	.7660	**50°**
10′	.6450	.7642	50′
20′	.6472	.7623	40′
30′	.6494	.7604	30′
40′	.6517	.7585	20′
50′	.6539	.7566	10′
41°	.6561	.7547	**49°**
10′	.6583	.7528	50′
20′	.6604	.7509	40′
30′	.6626	.7490	30′
40′	.6648	.7470	20′
50′	.6670	.7451	10′
42°	.6691	.7431	**48°**
10′	.6713	.7412	50′
20′	.6734	.7392	40′
30′	.6756	.7373	30′
40′	.6777	.7353	20′
50′	.6799	.7333	10′
43°	.6820	.7314	**47°**
10′	.6841	.7294	50′
20′	.6862	.7274	40′
30′	.6884	.7254	30′
40′	.6905	.7234	20′
50′	.6926	.7214	10′
44°	.6947	.7193	**46°**
10′	.6967	.7173	50′
20′	.6988	.7153	40′
30′	.7009	.7133	30′
40′	.7030	.7112	20′
50′	.7050	.7092	10′
45°	.7071	.7071	**45°**
	cos θ	sin θ	θ

φ	tan φ	ctn φ	
0°	.000000		90°
10′	.002909		50′
20′	.005818		40′
30′	.008727		30′
40′	.011636		20′
50′	.014545		10′
1°	.017455	57.	89°
10′	.02036	49.	50′
20′	.02328	43.	40′
30′	.02619	38.	30′
40′	.02910	34.	20′
50′	.03201	31.	10′
2°	.03492	28.6	88°
10′	.03783	26.4	50′
20′	.04075	24.5	40′
30′	.04366	22.9	30′
40′	.04658	21.5	20′
50′	.04949	20.2	10′
3°	.05241	19.1	87°
10′	.05533	18.1	50′
20′	.05824	17.2	40′
30′	.06116	16.3	30′
40′	.06408	15.6	20′
50′	.06700	14.9	10′
4°	.06993	14.3	86°
10′	.07285	13.73	50′
20′	.07578	13.20	40′
30′	.07870	12.71	30′
40′	.08163	12.25	20′
50′	.08456	11.83	10′
5°	.08749	11.43	85°
10′	.09042	11.06	50′
20′	.09335	10.71	40′
30′	.09629	10.39	30′
40′	.09923	10.08	20′
50′	.10216	9.79	10′
6°	.10510	9.51	84°
10′	.1080	9.26	50′
20′	.1110	9.01	40′
30′	.1139	8.78	30′
40′	.1169	8.56	20′
50′	.1198	8.34	10′
7°	.1228	8.14	83°
10′	.1257	7.95	50′
20′	.1287	7.77	40′
30′	.1317	7.60	30′
			82°
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
7°			
30′	.1317	7.60	30′
40′	.1346	7.43	20′
50′	.1376	7.27	10′
8°	.1405	7.12	82°
10′	.1435	6.97	50′
20′	.1465	6.83	40′
30′	.1495	6.69	30′
40′	.1524	6.56	20′
50′	.1554	6.43	10′
9°	.1584	6.31	81°
10′	.1614	6.197	50′
20′	.1644	6.084	40′
30′	.1673	5.976	30′
40′	.1703	5.871	20′
50′	.1733	5.769	10′
10°	.1763	5.671	80°
10′	.1793	5.576	50′
20′	.1823	5.485	40′
30′	.1853	5.396	30′
40′	.1883	5.309	20′
50′	.1914	5.226	10′
11°	.1944	5.145	79°
10′	.1974	5.066	50′
20′	.2004	4.989	40′
30′	.2035	4.915	30′
40′	.2065	4.843	20′
50′	.2095	4.773	10′
12°	.2126	4.705	78°
10′	.2156	4.638	50′
20′	.2186	4.574	40′
30′	.2217	4.511	30′
40′	.2247	4.449	20′
50′	.2278	4.390	10′
13°	.2309	4.331	77°
10′	.2339	4.275	50′
20′	.2370	4.219	40′
30′	.2401	4.165	30′
40′	.2432	4.113	20′
50′	.2462	4.061	10′
14°	.2493	4.011	76°
10′	.2524	3.962	50′
20′	.2555	3.914	40′
30′	.2586	3.867	30′
40′	.2617	3.821	20′
50′	.2648	3.776	10′
15°	.2679	3.732	75°
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
15°	.2679	3.732	75°
10′	.2711	3.689	50′
20′	.2742	3.647	40′
30′	.2773	3.606	30′
40′	.2805	3.566	20′
50′	.2836	3.526	10′
16°	.2867	3.487	74°
10′	.2899	3.450	50′
20′	.2931	3.412	40′
30′	.2962	3.376	30′
40′	.2994	3.340	20′
50′	.3026	3.305	10′
17°	.3057	3.271	73°
10′	.3089	3.237	50′
20′	.3121	3.204	40′
30′	.3153	3.172	30′
40′	.3185	3.140	20′
50′	.3217	3.108	10′
18°	.3249	3.078	72°
10′	.3281	3.047	50′
20′	.3314	3.018	40′
30′	.3346	2.989	30′
40′	.3378	2.960	20′
50′	.3411	2.932	10′
19°	.3443	2.904	71°
10′	.3476	2.877	50′
20′	.3508	2.850	40′
30′	.3541	2.824	30′
40′	.3574	2.798	20′
50′	.3607	2.773	10′
20°	.3640	2.747	70°
10′	.3673	2.723	50′
20′	.3706	2.699	40′
30′	.3739	2.675	30′
40′	.3772	2.651	20′
50′	.3805	2.628	10′
21°	.3839	2.605	69°
10′	.3872	2.583	50′
20′	.3906	2.560	40′
30′	.3939	2.539	30′
40′	.3973	2.517	20′
50′	.4006	2.496	10′
22°	.4040	2.475	68°
10′	.4074	2.455	50′
20′	.4108	2.434	40′
30′	.4142	2.414	30′
			67°
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
22°			
30′	.4142	2.414	30′
40′	.4176	2.394	20′
50′	.4210	2.375	10′
23°	.4245	2.356	**67°**
10′	.4279	2.337	50′
20′	.4314	2.318	40′
30′	.4348	2.300	30′
40′	.4383	2.282	20′
50′	.4417	2.264	10′
24°	.4452	2.246	**66°**
10′	.4487	2.229	50′
20′	.4522	2.211	40′
30′	.4557	2.194	30′
40′	.4592	2.177	20′
50′	.4628	2.161	10′
25°	.4663	2.145	**65°**
10′	.4699	2.128	50′
20′	.4734	2.112	40′
30′	.4770	2.097	30′
40′	.4806	2.081	20′
50′	.4841	2.066	10′
26°	.4877	2.050	**64°**
10′	.4913	2.035	50′
20′	.4950	2.020	40′
30′	.4986	2.006	30′
40′	.5022	1.991	20′
50′	.5059	1.977	10′
27°	.5095	1.963	**63°**
10′	.5132	1.949	50′
20′	.5169	1.935	40′
30′	.5206	1.921	30′
40′	.5243	1.907	20′
50′	.5280	1.894	10′
28°	.5317	1.881	**62°**
10′	.5354	1.868	50′
20′	.5392	1.855	40′
30′	.5430	1.842	30′
40′	.5467	1.829	20′
50′	.5505	1.816	10′
29°	.5543	1.804	**61°**
10′	.5581	1.792	50′
20′	.5619	1.780	40′
30′	.5658	1.767	30′
40′	.5696	1.756	20′
50′	.5735	1.744	10′
30°	.5774	1.732	**60°**
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
30°	.5774	1.732	**60°**
10′	.5812	1.720	50′
20′	.5851	1.709	40′
30′	.5890	1.698	30′
40′	.5930	1.686	20′
50′	.5969	1.675	10′
31°	.6009	1.664	**59°**
10′	.6048	1.653	50′
20′	.6088	1.643	40′
30′	.6128	1.632	30′
40′	.6168	1.621	20′
50′	.6208	1.611	10′
32°	.6249	1.600	**58°**
10′	.6289	1.590	50′
20′	.6330	1.580	40′
30′	.6371	1.570	30′
40′	.6412	1.560	20′
50′	.6453	1.550	10′
33°	.6494	1.540	**57°**
10′	.6536	1.530	50′
20′	.6577	1.520	40′
30′	.6619	1.511	30′
40′	.6661	1.501	20′
50′	.6703	1.492	10′
34°	.6745	1.483	**56°**
10′	.6787	1.473	50′
20′	.6830	1.464	40′
30′	.6873	1.455	30′
40′	.6916	1.446	20′
50′	.6959	1.437	10′
35°	.7002	1.428	**55°**
10′	.7046	1.419	50′
20′	.7089	1.411	40′
30′	.7133	1.402	30′
40′	.7177	1.393	20′
50′	.7221	1.385	10′
36°	.7265	1.376	**54°**
10′	.7310	1.368	50′
20′	.7355	1.360	40′
30′	.7400	1.351	30′
40′	.7445	1.343	20′
50′	.7490	1.335	10′
37°	.7536	1.327	**53°**
10′	.7581	1.319	50′
20′	.7627	1.311	40′
30′	.7673	1.303	30′
			52°
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
37°			
30′	.7673	1.303	30′
40′	.7720	1.295	20′
50′	.7766	1.288	10′
38°	.7813	1.280	**52°**
10′	.7860	1.272	50′
20′	.7907	1.265	40′
30′	.7954	1.257	30′
40′	.8002	1.250	20′
50′	.8050	1.242	10′
39°	.8098	1.235	**51°**
10′	.8146	1.228	50′
20′	.8195	1.220	40′
30′	.8243	1.213	30′
40′	.8292	1.206	20′
50′	.8342	1.199	10′
40°	.8391	1.192	**50°**
10′	.8441	1.185	50′
20′	.8491	1.178	40′
30′	.8541	1.171	30′
40′	.8591	1.164	20′
50′	.8642	1.157	10′
41°	.8693	1.150	**49°**
10′	.8744	1.144	50′
20′	.8796	1.137	40′
30′	.8847	1.130	30′
40′	.8899	1.124	20′
50′	.8952	1.117	10′
42°	.9004	1.111	**48°**
10′	.9057	1.104	50′
20′	.9110	1.098	40′
30′	.9163	1.091	30′
40′	.9217	1.085	20′
50′	.9271	1.079	10′
43°	.9325	1.072	**47°**
10′	.9380	1.066	50′
20′	.9435	1.060	40′
30′	.9490	1.054	30′
40′	.9545	1.048	20′
50′	.9601	1.042	10′
44°	.9657	1.036	**46°**
10′	.9713	1.030	50′
20′	.9770	1.024	40′
30′	.9827	1.018	30′
40′	.9884	1.012	20′
50′	.9942	1.006	10′
45°	1.0000	1.000	**45°**
	ctn θ	tan θ	θ

Natural Secants and Cosecants.

ϕ	$\sec\phi$	$\csc\phi$	
0°	1.0000		**90°**
10′	1.0000		50′
20′	1.0000		40′
30′	1.0000		30′
40′	1.0001		20′
50′	1.0001		10′
1°	1.0002	57.	**89°**
10′	1.0002	49.	50′
20′	1.0003	43.	40′
30′	1.0003	38.	30′
40′	1.0004	34.	20′
50′	1.0005	31.	10′
2°	1.0006	28.7	**88°**
10′	1.0007	26.5	50′
20′	1.0008	24.6	40′
30′	1.0010	22.9	30′
40′	1.0011	21.5	20′
50′	1.0012	20.2	10′
3°	1.0014	19.1	**87°**
10′	1.0015	18.1	50′
20′	1.0017	17.2	40′
30′	1.0019	16.4	30′
40′	1.0021	15.6	20′
50′	1.0022	15.0	10′
4°	1.0024	14.3	**86°**
10′	1.0026	13.76	50′
20′	1.0029	13.23	40′
30′	1.0031	12.75	30′
40′	1.0033	12.29	20′
50′	1.0036	11.87	10′
5°	1.0038	11.47	**85°**
10′	1.0041	11.10	50′
20′	1.0043	10.76	40′
30′	1.0046	10.43	30′
40′	1.0049	10.13	20′
50′	1.0052	9.84	10′
6°	1.0055	9.57	**84°**
10′	1.0058	9.31	50′
20′	1.0061	9.07	40′
30′	1.0065	8.83	30′
40′	1.0068	8.61	20′
50′	1.0072	8.40	10′
7°	1.0075	8.21	**83°**
10′	1.0079	8.02	50′
20′	1.0082	7.83	40′
30′	1.0086	7.66	30′
			82°
	$\csc\theta$	$\sec\theta$	θ

ϕ	$\sec\phi$	$\csc\phi$	
7°			
30′	1.009	7.66	30′
40′	1.009	7.50	20′
50′	1.009	7.34	10′
8°	1.010	7.19	**82°**
10′	1.010	7.04	50′
20′	1.011	6.90	40′
30′	1.011	6.77	30′
40′	1.012	6.64	20′
50′	1.012	6.51	10′
9°	1.012	6.39	**81°**
10′	1.013	6.277	50′
20′	1.013	6.166	40′
30′	1.014	6.059	30′
40′	1.014	5.955	20′
50′	1.015	5.855	10′
10°	1.015	5.759	**80°**
10′	1.016	5.665	50′
20′	1.016	5.575	40′
30′	1.017	5.487	30′
40′	1.018	5.403	20′
50′	1.018	5.320	10′
11°	1.019	5.241	**79°**
10′	1.019	5.164	50′
20′	1.020	5.089	40′
30′	1.020	5.016	30′
40′	1.021	4.945	20′
50′	1.022	4.876	10′
12°	1.022	4.810	**78°**
10′	1.023	4.745	50′
20′	1.024	4.682	40′
30′	1.024	4.620	30′
40′	1.025	4.560	20′
50′	1.026	4.502	10′
13°	1.026	4.445	**77°**
10′	1.027	4.390	50′
20′	1.028	4.336	40′
30′	1.028	4.284	30′
40′	1.029	4.232	20′
50′	1.030	4.182	10′
14°	1.031	4.134	**76°**
10′	1.031	4.086	50′
20′	1.032	4.039	40′
30′	1.033	3.994	30′
40′	1.034	3.950	20′
50′	1.034	3.906	10′
15°	1.035	3.864	**75°**
	$\csc\theta$	$\sec\theta$	θ

ϕ	$\sec\phi$	$\csc\phi$	
15°	1.035	3.864	**75°**
10′	1.036	3.822	50′
20′	1.037	3.782	40′
30′	1.038	3.742	30′
40′	1.039	3.703	20′
50′	1.039	3.665	10′
16°	1.040	3.628	**74°**
10′	1.041	3.592	50′
20′	1.042	3.556	40′
30′	1.043	3.521	30′
40′	1.044	3.487	20′
50′	1.045	3.453	10′
17°	1.046	3.420	**73°**
10′	1.047	3.388	50′
20′	1.048	3.356	40′
30′	1.049	3.326	30′
40′	1.049	3.295	20′
50′	1.050	3.265	10′
18°	1.051	3.236	**72°**
10′	1.052	3.207	50′
20′	1.053	3.179	40′
30′	1.054	3.152	30′
40′	1.056	3.124	20′
50′	1.057	3.098	10′
19°	1.058	3.072	**71°**
10′	1.059	3.046	50′
20′	1.060	3.021	40′
30′	1.061	2.996	30′
40′	1.062	2.971	20′
50′	1.063	2.947	10′
20°	1.064	2.924	**70°**
10′	1.065	2.901	50′
20′	1.066	2.878	40′
30′	1.068	2.855	30′
40′	1.069	2.833	20′
50′	1.070	2.812	10′
21°	1.071	2.790	**69°**
10′	1.072	2.769	50′
20′	1.074	2.749	40′
30′	1.075	2.729	30′
40′	1.076	2.709	20′
50′	1.077	2.689	10′
22°	1.079	2.669	**68°**
10′	1.080	2.650	50′
20′	1.081	2.632	40′
30′	1.082	2.613	30′
			67°
	$\csc\theta$	$\sec\theta$	θ

φ	sec φ	csc φ	
22°			
30′	1.082	2.613	30′
40′	1.084	2.595	20′
50′	1.085	2.577	10′
23°	1.086	2.559	**67°**
10′	1.088	2.542	50′
20′	1.089	2.525	40′
30′	1.090	2.508	30′
40′	1.092	2.491	20′
50′	1.093	2.475	10′
24°	1.095	2.459	**66°**
10′	1.096	2.443	50′
20′	1.097	2.427	40′
30′	1.099	2.411	30′
40′	1.100	2.396	20′
50′	1.102	2.381	10′
25°	1.103	2.366	**65°**
10′	1.105	2.352	50′
20′	1.106	2.337	40′
30′	1.108	2.323	30′
40′	1.109	2.309	20′
50′	1.111	2.295	10′
26°	1.113	2.281	**64°**
10′	1.114	2.268	50′
20′	1.116	2.254	40′
30′	1.117	2.241	30′
40′	1.119	2.228	20′
50′	1.121	2.215	10′
27°	1.122	2.203	**63°**
10′	1.124	2.190	50′
20′	1.126	2.178	40′
30′	1.127	2.166	30′
40′	1.129	2.154	20′
50′	1.131	2.142	10′
28°	1.133	2.130	**62°**
10′	1.134	2.118	50′
20′	1.136	2.107	40′
30′	1.138	2.096	30′
40′	1.140	2.085	20′
50′	1.142	2.074	10′
29°	1.143	2.063	**61°**
10′	1.145	2.052	50′
20′	1.147	2.041	40′
30′	1.149	2.031	30′
40′	1.151	2.020	20′
50′	1.153	2.010	10′
30°	1.155	2.000	**60°**
	csc θ	sec θ	θ

φ	sec φ	csc φ	
30°	1.155	2.000	**60°**
10′	1.157	1.990	50′
20′	1.159	1.980	40′
30′	1.161	1.970	30′
40′	1.163	1.961	20′
50′	1.165	1.951	10′
31°	1.167	1.942	**59°**
10′	1.169	1.932	50′
20′	1.171	1.923	40′
30′	1.173	1.914	30′
40′	1.175	1.905	20′
50′	1.177	1.896	10′
32°	1.179	1.887	**58°**
10′	1.181	1.878	50′
20′	1.184	1.870	40′
30′	1.186	1.861	30′
40′	1.188	1.853	20′
50′	1.190	1.844	10′
33°	1.192	1.836	**57°**
10′	1.195	1.828	50′
20′	1.197	1.820	40′
30′	1.199	1.812	30′
40′	1.202	1.804	20′
50′	1.204	1.796	10′
34°	1.206	1.788	**56°**
10′	1.209	1.781	50′
20′	1.211	1.773	40′
30′	1.213	1.766	30′
40′	1.216	1.758	20′
50′	1.218	1.751	10′
35°	1.221	1.743	**55°**
10′	1.223	1.736	50′
20′	1.226	1.729	40′
30′	1.228	1.722	30′
40′	1.231	1.715	20′
50′	1.233	1.708	10′
36°	1.236	1.701	**54°**
10′	1.239	1.695	50′
20′	1.241	1.688	40′
30′	1.244	1.681	30′
40′	1.247	1.675	20′
50′	1.249	1.668	10′
37°	1.252	1.662	**53°**
10′	1.255	1.655	50′
20′	1.258	1.649	40′
30′	1.260	1.643	30′
			52°
	csc θ	sec θ	θ

φ	sec φ	csc φ	
37°			
30′	1.260	1.643	30′
40′	1.263	1.636	20′
50′	1.266	1.630	10′
38°	1.269	1.624	**52°**
10′	1.272	1.618	50′
20′	1.275	1.612	40′
30′	1.278	1.606	30′
40′	1.281	1.601	20′
50′	1.284	1.595	10′
39°	1.287	1.589	**51°**
10′	1.290	1.583	50′
20′	1.293	1.578	40′
30′	1.296	1.572	30′
40′	1.299	1.567	20′
50′	1.302	1.561	10′
40°	1.305	1.556	**50°**
10′	1.309	1.550	50′
20′	1.312	1.545	40′
30′	1.315	1.540	30′
40′	1.318	1.535	20′
50′	1.322	1.529	10′
41°	1.325	1.524	**49°**
10′	1.328	1.519	50′
20′	1.332	1.514	40′
30′	1.335	1.509	30′
40′	1.339	1.504	20′
50′	1.342	1.499	10′
42°	1.346	1.494	**48°**
10′	1.349	1.490	50′
20′	1.353	1.485	40′
30′	1.356	1.480	30′
40′	1.360	1.476	20′
50′	1.364	1.471	10′
43°	1.367	1.466	**47°**
10′	1.371	1.462	50′
20′	1.375	1.457	40′
30′	1.379	1.453	30′
40′	1.382	1.448	20′
50′	1.386	1.444	10′
44°	1.390	1.440	**46°**
10′	1.394	1.435	50′
20′	1.398	1.431	40′
30′	1.402	1.427	30′
40′	1.406	1.423	20′
50′	1.410	1.418	10′
45°	1.414	1.414	**45°**
	csc θ	sec θ	θ

EXPLANATION OF THE TABLES.

§ 1. TABLES IN GENERAL.

a. One quantity is said to be a **function** of another, when the former quantity is regarded as determined by the latter, according to some rule or formula. E. g. x^2, $\sqrt{x}$, $\log x$, $\sin x$, $\log \sin x$, are all called functions of x. A **mathematical table** is an orderly arrangement of the values of some function for certain selected values of the quantity by which it is regarded as determined. The successive values of the latter quantity are assumed arbitrarily, and generally at equal intervals; and this quantity is called the **argument** of the table. Some functions require several independent quantities for their determination; and the corresponding tables are tables of several arguments. Thus, a multiplication-table is a table of *two* arguments; namely, the two factors.

b. A table may be used in two ways: **directly** and **inversely**. The direct use of the table consists in finding the value of the function for an assumed value of the argument; the inverse use, in finding the value of the argument for an assumed value of the function.

c. Before beginning to use any table, the student should give attentive consideration to its arrangement, and to the best mode of employing it with accuracy and ease. Every feature of it should be carefully examined, and the explanations which are attached to it should be fully mastered. The time thus spent will be time gained, contributing not only to power in computation, but also, very materially, to the thorough practical knowledge of the nature of the tabulated functions.

§ 2. INTERPOLATION.

a. **Interpolation** consists in finding the value of one of the two quantities, argument and function, for an assumed value of the other quantity, lying between two successive tabulated values. Most mathematical tables are so constructed as to admit of interpolation by the principle that *corresponding non-tabulated values of the function and argument lie between corresponding tabulated values and divide the differences between them in the same ratio.* This is the principle of **proportional parts**. Let x_1 and x_2 be two successive tabulated values of the argument of a table, and u_1 and u_2 the correspond-

ing values of the function. Then, $x_2 - x_1$ and $u_2 - u_1$ are called corresponding tabular differences. We shall denote these differences by Δx and Δu. If, now, x and u are corresponding values of the function and argument, of which one is known to lie between the two above-cited tabulated values of the same quantity, the principle of proportional parts is that if

$$\lambda = \frac{x - x_1}{\Delta x}, \qquad \lambda' = \frac{x_2 - x}{\Delta x} = 1 - \lambda.$$

$$\mu = \frac{u - u_1}{\Delta u}, \qquad \mu' = \frac{u_2 - u}{\Delta u} = 1 - \mu,$$

then (to the limit of accuracy belonging to the table)

$$\lambda = \mu, \qquad \lambda' = \mu',$$

or,

$$u = u_1 + \lambda \Delta u = u_2 - \lambda' \Delta u,$$
$$x = x_1 + \mu \Delta x = x_2 - \mu' \Delta x.$$

Thus, the required value of the function or argument may be obtained by applying a **correction** to either of the two tabulated values between which the required value lies. In computing this correction, the *signs* of the differences employed must be carefully observed. If x_1 and x_2 are so chosen as to make Δx positive, Δu may be either positive or negative. In the former case, the function is said to be *increasing*; in the latter, *decreasing*.

b. Either of the two formulas given above for finding u may be employed, in interpolation, in the direct use of the table; either of the formulas for x may be employed in the inverse use of the table. In most tables, $\Delta x =$ *one unit* in the last numeral place of the tabulated values of x. Hence λ is composed of *the figures which follow that numeral place* in the given non-tabulated value of the argument, preceded by a decimal-point; while λ' is the *complement* of λ (that is, can be found by subtracting from 9 each figure of λ except the last, and subtracting that from 10). The correction for u is, therefore, found simply by *multiplying* the figures in question into Δu, and pointing off according to the case; x will be corrected by *annexing* to x_1 the figures of $\frac{u - u_1}{\Delta u}$, or the figures complementary to $\frac{u_2 - u}{\Delta u}$.

c. In some of the tables of this collection will be found, set against each value of the function, *a number in small type*, which shows what Δu would be if the function varied through a whole interval corresponding to Δx at the same rate at which it is changing when it passes through the value against which this number is set. This number may be called the *rate of difference*, or simply the *difference*, of u, and may be substituted for Δu in the formulas of interpolation. But, in that case, we ought to work from the NEAREST tabulated value of x or u; that is, from x_1 or u_1 when λ or $\mu < 0.5$, and from x_2 or u_2 when λ' or $\mu' < 0.5$. (See examples in the explanation of the table of *Logarithms of Circular Functions*.)

d. An interpolated value of the function should not be carried out beyond the last numeral place of the tabulated value from which it is computed; so that, in finding $\lambda \Delta u$ or $\lambda' \Delta u$, we should reject the decimal part of the product, Δu being regarded as an integer. Owing to the combination of the figures rejected in the correction and those omitted in the tabulated value of the function, an interpolated value is liable to an error of ± 1 in the last figure.

The number of figures annexed to the tabulated value of the argument, in inverse interpolation, should be less by one than the number of figures contained in Δu. It is sometimes, indeed, made *equal* to the latter number (and will always be, if Δu consists of only one figure); but, in that case, the *last figure* must be regarded as *uncertain.* When the given value of the function is the result of computation, of course this uncertainty may extend back to earlier figures.

e. In taking the correction of either the function or the argument only to a certain number of figures, we must observe the following rule, which is a universal rule of computation:—

Whenever figures are neglected at the end of a number, if the figures neglected amount to more than half a unit in the place of the last figure retained, the last figure retained must be increased by 1. E. g. 27.528 = 27.53 to the nearest hundredth = 27.5 to the nearest tenth = 28 to the nearest unit = 30 to the nearest ten.

f. The various rules of interpolation will be found to be fully exemplified below, in the explanations of the tables of *Logarithms* and *Logarithms of Circular Functions.*

g. In interpolating in some tables (e. g. in VLACQ's great ten-place table), we must have regard to **second differences**, or differences between differences. In this case, we add to the above formulas for u the term

$$-\tfrac{1}{2}\,\lambda\lambda'\,\Delta^2 u,$$

where $\Delta^2 u$ denotes the second difference of u, taken positively when Δu is increasing. The greatest value of this term is *one eighth* of $\Delta^2 u$, so that it is insignificant when $\Delta^2 u < 4$. *In the present tables this term may always be neglected;* although it is useful as measuring the extent of error, and may occasionally guide the judgment of the computer when the fractional part of the correction is equal, or nearly equal, to 0.5. But where such nicety of work seems to be called for, it is best to use a table of a larger number of places.

§ 3. PROPORTIONAL PARTS.

a. The table of **Proportional Parts** (folded page) may be used in connexion with any other table, as an aid in *interpolation.* It contains the product of every integer from 1 to 100 by every *tenth* from 0.1 to 0.9. If the multiplier consists of one figure in any other numeral place, it is only necessary to change the position of the decimal-point in the product. To multiply a number of two figures by any decimal whatever, we must find the products which correspond to the successive figures of the multiplier, and add them together. The decimal part of the result is generally to be discarded, and in that case the general rule given above (in § 2, *e*) must be observed. Thus, let it be required to find 0.619 × 37. Looking in the column belonging to 37, we find

$$\begin{aligned} 0.6 \times 37 &= 22.2 \\ 0.01 \times 37 &= 0.37 \\ 0.009 \times 37 &= \underline{0.333} \\ \therefore\ 0.619 \times 37 &= 23. \end{aligned}$$

In like manner, we find

$$0.27 \times 15 = 4, \quad 0.59 \times 73 = 43, \quad 0.78 \times 69 = 54,$$
$$0.96 \times 84 = 81, \quad 0.36 \times 57 = 21, \quad 0.289 \times 51 = 15,$$
$$0.483 \times 93 = 45, \quad 0.374 \times 82 = 31, \quad 0.053 \times 68 = 4.$$

b. This table can also be used *inversely*. Thus, let it be required to find, to two decimal-places, what part 36 is of 79. Looking in the column of 79, we find

$$\begin{array}{r} 36 \\ 0.4 \times 79 = \underline{31.6} \\ 4.4 \end{array}$$

$$0.06 \times 79 = 4.74 \text{ (the \textit{nearest} product)}$$

$$\therefore \frac{36}{79} = 0.46.$$

In like manner, we find

$$\frac{29}{68} = 0.43, \quad \frac{72}{89} = 0.81, \quad \frac{31}{98} = 0.32, \quad \frac{26}{71} = 0.37, \quad \frac{45}{57} = 0.79, \quad \frac{11}{37} = 0.30.$$

A little practice will enable the student to use this table easily and rapidly.

§ 4. LOGARITHMS.

a. **Denary**, or **Briggsian**, logarithms, being those employed in actual computation, are always referred to, in this collection of tables, when the term *logarithm* is used without qualification. The **characteristic**, or integral part, of the denary logarithm of a number depends only on the position of the first significant figure of the number relatively to the units' place, and may be found by a well-known rule; the **mantissa**, or fractional part, depends only on the series of significant figures which compose the number, and is the only part of the logarithm for which it is necessary to employ a table. A table of logarithms is complete, to an assigned number of places, if it gives (explicitly or by interpolation) to that number of places the mantissa of the logarithm of every possible series of significant figures. Denary logarithms are, in general, incommensurable numbers, and cannot, therefore, be exactly expressed in figures. They are variously given, in different tables, to ten, seven, six, five, four, and three places of decimals. Four-place logarithms are sufficient for the ordinary purposes of engineering, navigation, the work of the physical and chemical laboratory, and many of the subordinate computations of astronomy; and, in most of these cases, are all that the accuracy of the data will justify us in using. Seven places are, however, needful for the more accurate kinds of astronomical and geodetic work.

b. If one number is the logarithm of another, the second number is called the **antilogarithm** of the first. This relation is denoted by the symbol $\log^{-1}$. Thus, if $u = \log x$, then $x = \log^{-1} u$. In an ordinary table of logarithms, the *argument* is the antilogarithm, which is tabulated to a greater or less number of figures, according to the number of places to

which the logarithm is given, and the *function* is the mantissa of the logarithm, which we often speak of simply as the *logarithm.*

To find the logarithm of any number.

c. If the number consists of *three significant figures*, seek the first two significant figures in the first column of the table of **Logarithms** (pp. 2, 3), and the third at the top of the table. In the line and column thus determined will be found the mantissa of the required logarithm, printed without the decimal-point. Find the characteristic by the rule, and prefix it, with the decimal-point, to the mantissa. E. g., log 2870 = 3.4579. If the given number has *less than three* significant figures, fill it out to three figures by anexing a zero or zeros. E. g., log 0.35 = log 0.350 = 9.5441 — 10, log 6 = log 6.00 = 0.7782. If the number has *more than three* significant figures, its logarithm must be found by one of the formulas of interpolation given above. The rule is: — *Find the logarithm of the first three significant figures of the given number and also that of the next following number of three figures* (1000 following 999); *then apply to* EITHER *of these two logarithms a correction, obtained by multiplying the difference between them by the difference between the given number and the three-figure number which corresponds to the logarithm chosen to be corrected, and rejecting* (*with due attention to the rule of* § 2, *e*) *as many figures at the end of the product as are contained in the latter difference.* The table of **Proportional Parts** may be employed in performing the multiplications. Thus, to find log 5668.4. Using the notation of the formulas of interpolation, and remembering that the place of the decimal-point in the given number may be disregarded in finding the mantissa of the required logarithm, we have

$$\begin{aligned} x_1 &= 566, & u_1 &= \text{mant} \log x_1 = 7528, \\ x_2 &= 567, & u_2 &= \text{mant} \log x_2 = 7536, \\ \Delta x &= \quad 1, & \Delta u &= \qquad\qquad\qquad\quad 8; \end{aligned}$$

so that log x may be found by either of the following methods: —

$$\lambda = 0.84, \qquad \lambda\,\Delta u = 6.72 = 7 \text{ to units}, \qquad u = 7528 + 7 = 7535;$$

or,

$$\lambda' = 1 - \lambda = 0.16, \qquad \lambda'\Delta u = 1.28 = 1 \text{ to units}, \qquad u = 7536 - 1 = 7535.$$

$$\therefore \log 5668.4 = 3.7535.$$

Let the beginner find the following logarithms by this method: —

log 59.43	= 1.7740,	log 0.0081472	= 7.9110 — 10,
log 284.8	= 2.4545,	log 572820	= 5.7581,
log 0.073748	= 8.8678 — 10,	log 0.50167	= 9.7004 — 10,
log 3.1607	= 0.4998,	log 99968.	= 4.9999.

The interpolated logarithm should never be carried to more than four decimal-places.

d. The work of interpolation may be shortened by using the column of proportional parts, marked P. P., on the right of the table. In using this column, one must work from the three-figure number NEAREST to the given

number (in the above example, from 567, not from 566). If the given number has only four figures, so that λ or λ' has only one figure, then the correction will be found in the column P. P., under λ or λ' (according as we are working from the number below or the number above the given number), and in the same line with the logarithm to be corrected. If the given number has more than four figures, the correction must be estimated by the observation of the corrections which correspond to the figures below and above the first figure of λ or λ'. E. g.

mant log 2848 = mant log 285 — cor. for .2 = 4548 — 3 = 4545;
mant log 56684 = mant log 567 — cor. for .16 = 7536 — 2 = 7534.

In the last case the correction is either 1 or 2, and, since .16 is nearer .20 than .10, we choose the correction belonging to .20. Larger tables show that the mantissa of the required logarithm, to five places, is 75346; so that the value found by the column P. P. is here nearly as accurate as that obtained by computation. There is a slightly greater liability to error when we use the column P. P. than when we interpolate by computation; but the disadvantage is generally insignificant. The last figure of an interpolated logarithm obtained from any table may always be one unit in error. E. g.: the true mant log of 57282 to five places is 75802; and this is a case in which the column P. P. gives a better result than computation.

The student is advised now to find all the logarithms in the above list by using the column P. P.

e. If the first figure of the given number is 1, it will be found tabulated to *four* figures in pp. 4, 5. The correction for a fifth and following figures may be found by the method of interpolation explained in *c*. As the differences are always small on these pages, and the corrections easily computed, the column P. P. is not here given; but, to facilitate taking the last difference, we have printed at the end of each line, under the heading 10, the first logarithm of the following line. Let the student find the following logarithms: —

log 11.737 = 1.0696, log 0.00100066 = 7.0003 — 10,
log 0.15703 = 9.1960 — 10, log 18597. = 4.2694.

To find the antilogarithm of any logarithm.

f. It is enough to explain the way of finding the series of significant figures which compose the antilogarithm, by means of the mantissa of the given logarithm; the pointing off of the antilogarithm being determined, according to rule, by the given characteristic. If the mantissa of the given logarithm is contained in the table, the required antilogarithm is at once found by inspection. Otherwise, we must resort to the formulas of interpolation, which give the following rule: — *Find two successive tabulated logarithms* (u_1 and u_2) *between which the given logarithm* (u) *lies; then divide the difference between either of these tabulated logarithms and the given logarithm* ($u - u_1$ or $u_2 - u$) *by the difference between the tabulated logarithms* (Δu), *carry out the quotient to the* NEAREST *tenth* (that is, to one figure, which may be 0), *and add it to or subtract it from the antilogarithm* (x_1 or x_2) *of the tabulated logarithm* (u_1 or u_2) *with which the given logarithm has been compared.* The antilogarithm is always a figure *annexed* to the three or four tabulated figures of x_1.

The division should not generally be carried beyond one figure. Even the first figure is, in most cases, somewhat uncertain. If the mantissa of the given logarithm is less than 3010, pp. 4, 5 should be used. On pp. 2, 3, the column P. P. may be employed.

Let it be required to find $\log^{-1} 1.5284$. We find

$$u_1 = 5276, \qquad x_1 = 337, \qquad u - u_1 = 8, \qquad u_2 - u = 5,$$

$$u_2 = 5289, \qquad x_2 = 338, \qquad \mu = \mu\,\Delta x = \frac{8}{13} = 0.6\ldots,$$

$$\Delta u = \;\; 13, \qquad \Delta x = \;\; 1, \qquad \mu' = \mu'\Delta x = \frac{5}{13} = 0.4\ldots,$$

$$x = 337 + 0.6 = 338 - 0.4 = 337.6;$$

$$\therefore \log^{-1} 1.5284 = 33.76.$$

More briefly, looking along the line of $u_2 = 5289$ for $5 = u_2 - u$ in column P. P., we find that 5 corresponds to the correction 4, which gives at once the required number. In like manner, the student may find

$$\log^{-1} 1.9155 = 82.32, \qquad \log^{-1} (5.8760 - 10) = 0.00007517,$$
$$\log^{-1} 3.8291 = 6747, \qquad \log^{-1} (9.5727 - 10) = 0.3738,$$
$$\log^{-1} 0.1548 = 1.4283, \qquad \log^{-1} (8.2731 - 10) = 0.018755.$$

g. The convenient usage of *making negative characteristics positive,* by the addition of 10, is followed, throughout the present collection of tables, whenever logarithms are printed with their characteristics. This must be always understood, though no explicit reference be made to it in the explanation of the table.

§ 5. LOGARITHMS OF SUMS AND DIFFERENCES.

a. This is one form of a table devised by Gauss to facilitate finding the logarithm of the *sum* or *difference* of two numbers which are themselves given only by their logarithms. The *argument* of the table is any logarithm, and may be called $\log x$; the *function* tabulated is then $\log (x + 1)$. It follows that, if the function is denoted by $\log x$, the argument is $\log (x - 1)$. The function may be called the **Gaussian** of the argument, and the argument the **anti-Gaussian** of the function; and the symbols $\mathfrak{G}$ and $\mathfrak{G}^{-1}$ may be used to denote these relations. Thus we have

$$\log (x + 1) = \mathfrak{G} \log x, \qquad \log (x - 1) = \mathfrak{G}^{-1} \log x.$$

b. **To find the Gaussian of a given logarithm.** Seek the characteristic of the given logarithm (increased by 10 if negative) at the top of the table, and the first two figures of the mantissa in the left-hand column. If the third and fourth figures of the mantissa are zero, the Gaussian will be found in the column and line thus determined; otherwise, it can be obtained by the method of interpolation which has been fully explained in § 2. In three columns of the table, the *rate of difference* of the Gaussian is printed in small type after the value of the function, and may be used instead of the tabular difference of the Gaussian *through half the tabular interval before and after* the value to which it is attached, as explained in

§ 2, *c*, and completely illustrated below, in the explanation of the table of *Logarithms of Circular Functions.* The table of Proportional Parts may be employed in computing the corrections. Examples: —

$$\mathfrak{G}\ 1.0960 = 1.1295, \qquad \mathfrak{G}\ (7.5265 - 10) = 0.0015,$$
$$\mathfrak{G}\ 3.8129 = 3.8130, \qquad \mathfrak{G}\ (9.6431 - 10) = 0.1582.$$

If $\log x < 6.0000 - 10$, $\quad \log(x + 1) = 0.0000$ to four places;
if $\log x > 4.0000$, $\quad \log(x + 1) = \log x$ to four places.

c. **To find the anti-Gaussian of a given logarithm.** Seek, in the body of the table, two successive logarithms between which the given logarithm lies, and then find the corresponding value of the argument by interpolation. Examples: —

$$\mathfrak{G}^{-1}\ 1.0960 = 1.0597, \qquad \mathfrak{G}^{-1}\ 0.1051 = 9.4373 - 10,$$
$$\mathfrak{G}^{-1}\ 3.8129 = 3.8128, \qquad \mathfrak{G}^{-1}\ 1.0216 = 0.9782.$$

d. **To find the logarithm of the sum or difference of the anti-logarithms of two given logarithms.** If m and n are two numbers,

$$m + n = n\left(\frac{m}{n} + 1\right), \qquad m - n = n\left(\frac{m}{n} - 1\right),$$

$$\log(m + n) = \log n + \log\left(\frac{m}{n} + 1\right) = \log n + \mathfrak{G} \log \frac{m}{n},$$

$$\log(m - n) = \log n + \log\left(\frac{m}{n} - 1\right) = \log n + \mathfrak{G}^{-1} \log \frac{m}{n}.$$

Example: —

Given $a = 4.142$, $b = 2.399$; to find $\sqrt{(a^2 + b^2)}$ and $\sqrt{(a^2 - b^2)}$.

$$\begin{array}{llll}
\log a & = 0.6172, & \log b & = 0.3800, \\
\log a^2 & = 1.2344, & \log b^2 & = 0.7600, \\
\log \frac{a^2}{b^2} & = 0.4744; & & \\
\mathfrak{G} \log \frac{a^2}{b^2} & = 0.6000, & \mathfrak{G}^{-1} \log \frac{a^2}{b^2} & = 0.2970, \\
\log b^2 & = 0.7600, & \log b^2 & = 0.7600, \\
\log (a^2 + b^2) & = 1.3600, & \log (a^2 - b^2) & = 1.0570, \\
\log \sqrt{(a^2 + b^2)} & = 0.6800, & \log \sqrt{(a^2 - b^2)} & = 0.5285, \\
\sqrt{(a^2 + b^2)} & = 4.787; & \sqrt{(a^2 - b^2)} & = 3.377.
\end{array}$$

§ 6. CIRCULAR, OR TRIGONOMETRIC, FUNCTIONS: NATURAL VALUES.

a. Three tables of the natural values of the trigonometric functions are given on pp. 22–27. Each table is broken up into six divisions, and occupies two pages. The argument is the angle, which is tabulated at intervals of 10′ from 0° to 90°. Angles in the *first half of the quadrant* will be found in the *left-hand column* of the several divisions of the table, and for those angles the names of the functions are to be taken from the *top* of the page; angles in the *second half of the quadrant* are to be found in the *right-hand*

column of the table, and for those angles the names of the functions are to be taken from the *bottom* of the page. The *angles* standing at the right and left in the same line are *complements* of each other; and the names of the *functions* at the top and bottom of the same column are *complementary*. The value of any of the functions for a non-tabulated angle, or the value of the angle for a non-tabulated value of one of the functions, can be found by the method of interpolation explained in § 2. The precepts of § 2, *d*, *e*, should be observed in computing the corrections. The tabulated values of the functions are generally given to four significant figures; but, in the tables of tangents and secants, they are sometimes given to a less number of figures (to avoid errors in interpolation), and are sometimes omitted altogether. In these cases the functions can be best found by finding their logarithms by the table of *Logarithms of Circular Functions* (see § 7), and then the numbers corresponding by the table of *Logarithms.*

b. To find any function of an angle *greater than* 90°, we must subtract from the given angle the greatest multiple of 90° which it contains; if an *even* multiple has been subtracted, we look out the required function of the remainder; if an *odd* multiple, the complementary function; and we then fix the *sign* of the function by considering the *quadrant* in which the given angle lies. For a *negative* angle, we find the required function of the corresponding positive angle, and then fix its sign by considering the quadrant of the angle.

c. Examples of the use of these tables: —

$$\begin{array}{llll}
\sin 77^\circ\, 37' = 0.9767, & \tan 53^\circ\, 04' = 1.330, & \sec 68^\circ\, 45' = 2.759, \\
\cos 16^\circ\, 19' = 0.9597, & \text{ctn}\ 3^\circ\, 18' = 17.4, & \csc 55^\circ\, 13' = 1.217;
\end{array}$$

$$\begin{array}{llll}
\sin 257^\circ\, 37' = -0.9767, & \tan 93^\circ\, 18' = -17.4, & \sec 325^\circ\, 13' = 1.217, \\
\cos 163^\circ\, 41' = -0.9597, & \text{ctn}\ 323^\circ\, 04' = -1.330, & \csc 158^\circ\, 45' = 2.759;
\end{array}$$

$$\begin{array}{llll}
\sin(-257^\circ\, 37') = 0.9767, & \tan(-93^\circ\, 18') = 17.4, & \sec(-325^\circ\, 13') = 1.217, \\
\cos(-163^\circ\, 41') = -0.9597, & \text{ctn}(-323^\circ\, 04') = 1.330, & \csc(-158^\circ\, 45') = -2.759;
\end{array}$$

$$\begin{array}{lllll}
\sin^{-1} 0.2000 & = 11^\circ\, 32' \text{ or } & = 168^\circ\, 28' \text{ or } & = 371^\circ\, 32', \text{ etc.,} \\
\cos^{-1}(-0.3542) & = 110^\circ\, 45' \text{ or } & = 249^\circ\, 15' \text{ or } & = 830^\circ\, 45', \text{ etc.,} \\
\tan^{-1}(-4.570) & = 102^\circ\, 21' \text{ or } & = 282^\circ\, 21' \text{ or } & = -77^\circ\, 39', \text{ etc.,} \\
\text{ctn}^{-1} 0.3163 & = 72^\circ\, 27' \text{ or } & = 252^\circ\, 27' \text{ or } & = -107^\circ\, 33', \text{ etc.,} \\
\sec^{-1} 5.000 & = 78^\circ\, 28' \text{ or } & = -78^\circ\, 28' \text{ or } & = \pm 281^\circ\, 32', \text{ etc.,} \\
\csc^{-1}(-3.529) & = -16^\circ\, 28' \text{ or } & = 196^\circ\, 28' \text{ or } & = -163^\circ\, 32', \text{ etc.}
\end{array}$$

§ 7. LOGARITHMS OF CIRCULAR FUNCTIONS.

To find the logarithm of any circular function of a given angle.

a. If the angle is *less than* 6°, the part of the table which occupies the upper half of p. 10 may be used. (See also *g.*) The *left-hand division* of this part of the table gives the values of a logarithm S (the characteristic and the first two figures of the mantissa being printed at the head of the column), with the angular limits between which each value may be used. Thus, for all positive angles less than 1° 51′.479, S = 6.4637; for all angles between 1° 51′.479 and 2° 49′.567, S = 6.4636; etc. The *next following*

division gives, in like manner, the values of a logarithm T. We must find *the logarithm of the angle*, reduced to minutes and decimals of a minute, and must then apply the formulas : —

$$\log \sin \phi = \log (\phi \text{ in minutes}) + S - 10,$$
$$\log \tan \phi = \log (\phi \text{ in minutes}) + T - 10.$$

The two right-hand divisions of this part of the table give the values of the log sec, with the angular limits for each value. The logarithms of the cosine, cotangent, and cosecant are the *arithmetical complements* (— 10) of the logarithms of the secant, tangent, and sine, respectively. Example : —

$l \sin 3° 15'.23 = 8.7541$, $l \tan 3° 15'.23 = 8.7548$, $l \sec 3° 15'.23 = 0.0007$,
$l \csc 3° 15'.23 = 1.2459$, $l \operatorname{ctn} 3° 15'.23 = 1.2452$, $l \cos 3° 15'.23 = 9.9993$;

the negative characteristics being here, as in the following examples, made positive by the addition of 10.

b. If the angle is *acute and greater than* 84°, we must take its *complement*, and then seek the *function complementary to that required*, for the angle thus obtained, by the method just expounded. Example : —

$l \sin 86° 44'.77 = 9.9993$, $l \tan 86° 44'.77 = 1.2452$, $l \sec 86° 44'.77 = 1.2459$,
$l \csc 86° 44'.77 = 0.0007$, $l \operatorname{ctn} 86° 44'.77 = 8.7548$, $l \cos 86° 44'.77 = 8.7541$.

c. If the angle is contained *between* 6° *and* 84°, we use the main part of the table, occupying the lower half of p. 10 and pp. 11–15. The angle is tabulated at intervals of 10′, from 6° to 45° in the left-hand column of the table, and from 45° to 84° in the right-hand column. The names of the functions are to be taken from the *tops* of the columns, when the angle is on the *left* ; and from the *bottoms* of the columns, when the angle is on the *right*. The angles on the right and left of any line and the names at the top and bottom of any column have the same relation to each other as in the tables of *Natural Values* (§ 6). The true characteristic in the first, third, and sixth columns is —1, but is printed 9. The six columns are arranged in pairs. The two functions in each pair of columns are *reciprocal* to each other ; and the logarithms are therefore *complementary*, and their differences are equal in value, with opposite signs. Down the middle of each double column are printed, in small type, the *rates of difference* of the logarithms in that double column. Each value of this rate may be used in interpolation, instead of Δu, *through half the interval before and after the line on which it stands*, as stated in § 2, *c*. Thus, in finding the logarithms of the circular functions of any angle between 25° 25′ and 25° 35′ we work from the values corresponding to 25° 30′, the *nearest* tabulated angle ; and compute the corrections by taking proportional parts of 26, 33, and 6, for the three pairs of functions. In applying the corrections, we must carefully observe, for each function, *whether the function is increasing or decreasing.*

For example, let the logarithms of the circular functions of 25° 27′.4 = 25° 30′ — 02′.6 be required. We find

$l \sin 25° 30' = 9.6340$	$l \tan 25° 30' = 9.6785$	$l \sec 25° 30' = 0.0445$
$0.26 \times 26 = \quad 7$	$0.26 \times 33 = \quad 9$	$0.26 \times 6 = \quad 2$
$l \sin 25° 27'.4 = 9.6333$,	$l \tan 25° 27'.4 = 9.6776$,	$l \sec 25° 27'.4 = 0.0443$,

l csc 25° 30′ = 0.3660	l ctn 25° 30′ = 0.3215	l cos 25° 30′ = 9.9555
7	9	2
l csc 25° 27′.4 = 0.3667,	l ctn 25° 27′.4 = 0.3224,	l cos 25° 27′.4 = 9.9557.

In like manner, we have

l sin 74° 46′ = 9.9845, l tan 74° 46′ = 0 5649, l sec 74° 46′ = 0.5804,
l csc 74° 46′ = 0.0155, l ctn 74° 46′ = 9.4351, l cos 74° 46′ = 9.4196.

d. If the angle is *greater than* 90°, or *negative*, we must use the method explained in § 6, *b*, for the tables of *Natural Values* of the circular functions. When the natural value of a circular function is negative, this should be indicated by writing the letter *n* after its logarithm. Examples:

l sin 105° 14′ = 9.9845, l tan 105° 14′ = 0 5649 *n*,
l sec 105° 14′ = 0.5804 *n*,

l csc 164° 46′ = 0.5804, l ctn 164° 46′ = 0.5649 *n*,
l cos 164° 46′ = 9.9845 *n*,

l sin (—74° 46′) = 9.9845 *n*, l tan (—105° 14′) = 0.5649,
l cos (—394° 46′) = 9.4196.

Given the logarithm of any circular function, to find the value of the corresponding angle.

e. If the given logarithm lies *without the limits of the main part of the table*, the upper part of p. 10 may be used. If the given logarithm is a *log sin less than* 9.0192, or a *log tan less than* 9.0216, subtract from it the proper value of S or T (or add the arithmetical complement), and the remainder is the log of the required angle in minutes. The limiting values of the log sin and log tan for each value of S and T are given in the table. If the given log is a *log csc greater than* 0.9808, or a *log ctn greater than* 0.9784, its arithmetical complement will be a log sin less than 9.0192, or a log tan less than 9.0216. If the given log is a *log sec less than* 0.0024, the limits between which the required angle lies are given by the table; the angle may have any value between these limits, and is not therefore very closely determined. If the given log is a *log cos greater than* 9.9976, its arithmetical complement is a log sec less than 0.0024.

If the given log is a *log sin, log tan, or log sec greater than* 9.9976, 0.9784, *or* 0.9808 (respectively), or a *log csc, log ctn, or log cos less than* 0.0024, 9.0216, *or* 9.0192 (respectively), we must change the name of the function to the *complementary name* (sin to cos, etc.), *find the corresponding angle* as above, and take the *complement* of the angle thus found. Examples: —

$(\log \sin)^{-1}$ 8.9542 = 5° 09′.8, $(\log \operatorname{ctn})^{-1}$ 2.0531 = 0° 30′.42,
$(\log \operatorname{ctn})^{-1}$ 9.0024 = 84° 15′.5, $(\log \sin)^{-1}$ 9.9983 = 84° 56′ $\pm 4\frac{1}{2}$′.

f. If the given logarithm is contained *within the limits of the main part of the table*, the required angle is found by ordinary interpolation; and we may use the printed rate of difference as the value of Δu, working in each case from the *nearest* tabulated value. The angle should be found to the *nearest minute*, or, when the difference exceeds 100, to the nearest *tenth* of a

minute. But in the right-hand pair of columns, the last figure of the angle thus found will generally be uncertain. Examples: —

Let it be required to find $(\log \sec)^{-1}\, 0.0643$; *i. e.* the angle of which the log sec is 0.0643. The nearest tabulated log sec is 0.0647. We have, then,

$$(\log \sec)^{-1}\, 0.0647 = 30°\, 30', \; u_2 - u = 4, \; \Delta u = 7, \; \frac{4}{7} = 0.6,$$

$$\therefore \; (\log \sec)^{-1}\, 0.0643 = 30°\, 30' - 06' = 30°\, 24'.$$

In like manner, let the student find

$(\log \sin)^{-1}\, 9.5663 = 21°\, 37'$, $(\log \text{ctn})^{-1}\, 0.0496 = 41°\, 44'$,
$(\log \cos)^{-1}\, 9.9188 = 33°\, 58'$ or $57'$, $(\log \sec)^{-1}\, 0.2272 = 53°\, 39'$,
$(\log \tan)^{-1}\, 0.7507 = 79°\, 56'$, $(\log \csc)^{-1}\, 0.1433 = 45°\, 58'$.

The angle may also be found by the next following table.

g. Pp. 8 and 9 may also be used for angles less than 6° or greater than 84°. E.g. $l \sin 4°\, 03'.4 = 8.8497$, $l \tan 4°\, 03'.4 = 8.8508$, $l \sec 4°\, 03'.4 = 0.0011$, $l \csc 4°\, 03'.4 = 1.1503$, $l \text{ ctn } 4°\, 03'.4 = 1.1492$, $l \cos 4°\, 03'.4 = 9.9989$.

§ 8. INVERSE CIRCULAR FUNCTIONS.

a. The table having this heading (pp. 16–18) is a table for finding the angle which corresponds to the given logarithm of a circular function. The logarithm (increased by 10) is the *argument* of the table, and is to be regarded as given to *four places* of decimals. It is tabulated at intervals of 0.0100 from 9.0000 to 0.0000 through the first page of the table, then at intervals of 0.0010, and in the last two divisions at intervals of 0.0001. The characteristic of the argument is printed at the head of the column. The figures supposed to follow the printed figures in the values of the argument are *zeros*. Thus, the first value is 9.0000, the next 9.0100, etc. The angle is given, for convenience of interpolation, in degrees and decimals of a degree. When found, it is easily reduced to degrees and minutes, if that is necessary, and should, in general, be taken *only to the nearest minute*. The angle under the heading $\sin^{-1} u$ is that angle of which the corresponding value of the argument, $\log u$, is the log sin ; etc. In interpolating in this table, we may use the *printed rate of difference* instead of Δu, working from the *nearest* tabulated value of the argument, and carefully observing whether the tabulated angle ought to be *increased* or *diminished*. *When the printed rate of difference is omitted*, this is because the interval is too great to admit of accurate interpolation. In this case, we must resort to those later divisions of the table in which the argument is tabulated at smaller intervals. *When the last figure of the tabulated angle is printed in small type*, this shows that that figure is uncertain, if the logarithm is given to only four places; that is, that there is a possible variation, on each side of the tabulated angle, as great as *half a unit* in the place of the figure so printed. For example, if $\log u = 9.9000$, we find the last figures of $\sin^{-1} u$ and $\cos^{-1} u$ to be printed in small type. Now, seven-place tables show that $(\log \sin)^{-1}$ $9.8999500 = 52°.581$, while $(\log \sin)^{-1}\, 9.9000500 = 52°.600$. But 9.9000 may represent any logarithm between these ; and hence the corresponding angle, in this case, admits a like variation, while $\cos^{-1} u$ may have any value between 37°.419 and 37°.400.

Neither of these difficulties presents itself in finding an angle from its log tan or log ctn. If $\log u = 9.9000$, $\tan^{-1} u$ can only vary from $38°.458$ to $38°.464$.

The angle found by interpolation should be carried out *only to the nearest hundredth of a degree, in any case.* The last column of the table shows that the angle is not always determined even to the nearest *tenth.*

b. If the characteristic of the given logarithm is 0, we must take its *arithmetical complement,* which will be the logarithm of the *reciprocal* function of the same angle. The angle can then be found by the table.

c. If the given logarithm is less than 9.0000, or greater than 0.0000, the tables in the upper part of p. 10 may be used, as explained in § 7, *e*; or pp. 8, 9.

d. Let us find by this table the angles sought above, in § 7, *f.* We have, in the case of the first example,

$$(\log \sec)^{-1}\, 0.0643 = (\log \cos)^{-1}\, 9.9357.$$

Then the table gives

$$\begin{aligned} (\log \cos)^{-1}\, 9.9360 &= 30°.35, \qquad \Delta u = 23,\\ 0.3 \times 0.23 \quad &= \quad .07 \\ \therefore\ (\log \cos)^{-1}\, 9.9357 &= 30°.42 = 30°\, 25'. \end{aligned}$$

In fact, the limits of the angle are $30°\, 24'.2$ and $30°\, 25'.6$, the mean value being $30°\, 24'.9$. In this case, the present table gives a better value than the other; but both values are admissible.

In like manner, we have

$$\begin{aligned} (\log \sin)^{-1}\, 9.5663 &= 21°.81 - \ \ 20 = 21°.61 = 21°\, 37',\\ (\log \cos)^{-1}\, 9.9188 &= 33°.92 + .04 = 33°.96 = 33°\, 58',\\ (\log \tan)^{-1}\, 0.7507 &= 79°.92 + .02 = 79°.94 = 79°\, 56',\\ (\log \mathrm{ctn})^{-1}\, 0.0496 &= 41°.71 + .03 = 41°.74 = 41°\, 44',\\ (\log \sec)^{-1}\, 0.2272 &= 53°.64 + .02 = 53°.66 = 53°\, 40',\\ (\log \csc)^{-1}\, 0.1433 &= 46°.01 - .04 = 45°.97 = 45°\, 58'. \end{aligned}$$

§ 9. HYPERBOLIC FUNCTIONS.

a. The **hyperbolic functions** are certain functions which bear relations to the equilateral hyperbola similar to those borne by the circular functions to the circle; and they may often be usefully employed both in computation and in analysis. They are named the hyperbolic sine, cosine, tangent, cotangent, secant, and cosecant; and are variously denoted by different writers. They are here represented by the symbols: Sh, Ch, Th, Cth, Sch, Csch. They may be defined by the following formulas, in which

$$\begin{aligned} \mathfrak{S} &= \text{the exponential base}\\ &= 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \cdots\\ &= 2.7182818285 \ldots : - \end{aligned}$$

$$\mathrm{Sh}\, x = \tfrac{1}{2}\left(\mathfrak{S}^{x} - \mathfrak{S}^{-x}\right), \qquad \mathrm{Ch}\, x = \tfrac{1}{2}\left(\mathfrak{S}^{x} + \mathfrak{S}^{-x}\right), \qquad \mathrm{Th}\, x = \frac{\mathrm{Sh}\, x}{\mathrm{Ch}\, x},$$

$$\mathrm{Cth}\, x = \frac{1}{\mathrm{Th}\, x}, \qquad \mathrm{Sch}\, x = \frac{1}{\mathrm{Ch}\, x}, \qquad \mathrm{Csch}\, x = \frac{1}{\mathrm{Sh}\, x}.$$

They bear to the circular functions the relations expressed by the following formulas, in which $i = \sqrt{-1}$: —

$$\mathrm{Sh}\, x = \frac{\sin xi}{i}, \qquad \sin x = \frac{\mathrm{Sh}\, xi}{i},$$
$$\mathrm{Ch}\, x = \cos xi, \qquad \cos x = \mathrm{Ch}\, xi,$$
$$\mathrm{Th}\, x = \frac{\tan xi}{i}, \qquad \tan x = \frac{\mathrm{Th}\, xi}{i},$$
$$\mathrm{Cth}\, x = i \operatorname{ctn} xi, \qquad \operatorname{ctn} x = i\, \mathrm{Cth}\, xi,$$
$$\mathrm{Sch}\, x = \sec xi, \qquad \sec x = \mathrm{Sch}\, xi,$$
$$\mathrm{Csch}\, x = i \csc xi, \qquad \csc x = i\, \mathrm{Csch}\, xi.$$

Again, if ϕ is so taken that

$$x = \text{nat log} \tan (45^\circ + \tfrac{1}{2}\phi),$$

$$\mathrm{Sh}\, x = \tan \phi, \qquad \mathrm{Ch}\, x = \sec \phi, \qquad \mathrm{Csch}\, x = \operatorname{ctn} \phi,$$
$$\mathrm{Th}\, x = \sin \phi, \qquad \mathrm{Sch}\, x = \cos \phi, \qquad \mathrm{Cth}\, x = \csc \phi.$$

The value of ϕ determined by this formula has been called by some writers the **Gudermannian** of x, and denoted by the symbol: gd x.

b. From $x = 0.00$ to $x = 1.00$, the function tabulated is gd x in degrees, at intervals of 0.01 in the value of x. The hyperbolic functions of x are then readily found, by the aid of the formulas last given, from the tables of circular functions. Beginning with $x = 1\,00$, log Sh x, log Ch x, and log Th x are tabulated, at intervals of 0.01 in the value of x, up to $x = 3.00$, the characteristic of each logarithm being placed at the head of its column; then at intervals of 0.1 up to $x = 6.0$; and lastly at intervals of 1 up to $x = 10.0$. The printed differences are to be used, as in other tables, each through *half the interval* before and after the line on which it stands.

c. If $x > 10$, log Th $x = 0.0000$, while log Sh x and log Ch x may be found by the formula and table given at the lower right-hand corner of p. 21. The quantity μ is the *modulus* of the denary system of logarithms; that is, it is the denary logarithm of the exponential base. The values of $n\mu$ being given for all integral values of n from 1 to 10, any product $x\mu$ is readily found, by adding together the products of μ by the successive figures of x. Only four decimal-figures should be retained in the result.

d. The functions log Cth x, log Sch x, and log Csch x are the *arithmetical complements* of log Th x, log Ch x, and log Sh x, respectively.

e. The table may be used both directly and inversely. Examples: —

$$\begin{aligned}
&\log \mathrm{Sh}\ 0.5378 &&= \log \tan 29^\circ.427 = 9.7513,\\
&\log \mathrm{Ch}\ 1.5280 &&= 0.3825,\\
&\log \mathrm{Cth}\ 1.8240 &&= 0.0226,\\
&\log \mathrm{Sh}\ 12.5913 &&= \log \mathrm{Ch}\ 12.5913 = 5.1673,\\
&(\log \mathrm{Ch})^{-1}\ 1.6000 &&= 4.377,\\
&(\log \mathrm{Sh})^{-1}\ 5.0000 &&= 12.206,\\
&(\log \mathrm{Th})^{-1}\ 9.9012 &&= 1.089,\\
&(\log \mathrm{Sch})^{-1}\ 9.5873 &&= 1.604,\\
&(\log \mathrm{Csch})^{-1}\ 0.3924 &&= \mathrm{gd}^{-1}\ (\log \tan)^{-1}\ 9.6076\\
&&&= \mathrm{gd}^{-1}\ 22^\circ.06 = 0.395.
\end{aligned}$$

§ 10. NATURAL LOGARITHMS.

a. The **natural** system of logarithms is that which is founded on the **exponential base** (see § 9). This number is defined as *the limiting value* to which the expression

$$(1+\epsilon)^{\frac{1}{\epsilon}} = \sqrt[\epsilon]{(1+\epsilon)}$$

approaches, as ϵ approaches 0. It is most frequently denoted by the letter e; but, as being one of the few peculiar constants of analysis, it is here represented by the symbol $\mathfrak{S}$, which may be read "**base.**"

The following formulas are proved in treatises on the Differential Calculus: —

$$\mathfrak{S} = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots,$$

$$\mathfrak{S}^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} + \dots,$$

$$\text{nat log }(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots;$$

the second formula being applicable to *all values* of x, but the last only when x is *numerically less than* 1. If x is *very small*, then *approximately*

$$\mathfrak{S}^x = 1 + x, \qquad \text{nat log }(1+x) = x, \qquad \text{nat log }(1-x) = -x.$$

We also have, in the natural system,

$$\log(a+h) = \log a + \log\left(1+\frac{h}{a}\right) = \log a + \frac{h}{a} - \frac{h^2}{2a^2} + \frac{h^3}{3a^3} - \dots,$$

provided h is numerically less than a.

The *rate of difference* of $\mathfrak{S}^x$, for $\Delta x = 1$, is always $\mathfrak{S}^x$, and that of nat log x is $\frac{1}{x}$.

b. The **numerical value** of $\mathfrak{S}$ or of any power of $\mathfrak{S}$ can be computed, to *any assigned number of decimal-places*, by using a sufficient number of terms of the first two series given above. Thus, to find $\mathfrak{S}$ to four decimal-places, we proceed as follows, observing that, if any term be divided by its number in the series, the next following term is obtained: —

$$\begin{array}{rl}
1) & 1.00000 \\
2) & 1.00000 \\
3) & 0.50000 \\
4) & 0.16667 \\
5) & 0.04167 \\
6) & 0.00833 \\
7) & 0.00139 \\
8) & 0.00020 \\
 & 0.00002 \\
\hline
\mathfrak{S} = & 2.7183 \;\dots\dots
\end{array}$$

c. The **modulus** of any system of logarithms is the logarithm of $\mathfrak{S}$ in that system. If m is the modulus of a system of which a is the base, then

$$a^m = \mathfrak{S}, \qquad \mathfrak{S}^{m^{-1}} = a.$$

The modulus of the *natural system itself* is 1. The values of the modulus of the *denary system* and of the reciprocal of that modulus are

$$\mu \;\; = \text{den log } \mathfrak{S} = 0.4342944819\ldots,$$
$$\mu^{-1} = \text{nat log } 10 = 2.3025850930\ldots.$$

By the rule for converting logarithms from one system to another, the logarithm of a number in any system may be found by *multiplying the modulus of that system into the natural logarithm of the same number.* Thus,

$$\text{den log } x = \mu \text{ nat log } x,$$
$$\text{nat log } x = \mu^{-1} \text{ den log } x.$$

By the aid of these formulas, the table at the bottom of p. 21 may be used to find the natural logarithm of any number, or the denary logarithm of any power of the exponential base, or to find a number from its natural logarithm. For example: —

$$\text{nat log } 72.5 \quad = 1.8603 \times \mu^{-1} = 4.2835,$$
$$\text{nat log } 1.0074 = 0.0032 \times \mu^{-1} = 0.0074,$$
$$\text{den log } \mathfrak{S}^{\frac{1}{7}} \quad = \tfrac{1}{7}\mu \qquad\qquad = 0.0620,$$

$$(\text{nat log})^{-1}\, 10.2108 = (\text{den log})^{-1}\, (10.2108 \times \mu)$$
$$= (\text{den log})^{-1}\, 4.4345 = 27194.$$

d. The natural system is so called, because, in the higher mathematics, it is convenient to regard all other systems as founded upon this. It is named by some writers **hyperbolic**, and by others **Neperian**. But in fact, it is not the system of Napier; nor has it any other relation to the hyperbola than that which belongs to logarithms in general.

e. We may make the following statement of the relation of logarithms and of the hyperbolic functions to the hyperbola, using the notation of Analytic Geometry: —

Let $xy = 1$ be the equation of an hyperbola referred to its asymptotes. It can be proved by the Integral Calculus that the *area*, contained between the curve and the axis of x, and between two ordinates of which one is drawn to the vertex of the curve, is measured by log x in the system of which the modulus is sin ω. Thus, *the logarithms belonging to any system may be represented by the areas of an appropriate hyperbola.* The natural system corresponds to the equilateral hyperbola, for which sin $\omega = 1$.

Again, if u denotes twice the area of the *sector* of the hyperbola $x^2 - y^2 = 1$, contained between the axis of x and a radius vector from the centre, then

$$x = \text{Ch } u, \qquad y = \text{Sh } u;$$

just as, in the circle $x^2 + y^2 = 1$, with a similar meaning of u,

$$x = \cos u, \qquad y = \sin u.$$

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